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Star Number Of A Graph

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Abstract

In this short paper, we considered simple graphs. We introduce the notions: star graph generated by a vertex and star number of a graph. We provide some examples. We proved that the star number of the LZDG(R)is 16 where LZDG(R) is the left zero divisor graph of the ring R and R is the 2×2-matrix ring over $\mathbb{Z}_2$.

Keywords: Graph, Degree of a vertex, Star graph, Star number, Star graph generated by vertex, Left zero divisor graph.
Mathematics Subject Classification: 05C20, 05C76, 05C99, 13E15, 68R10

1 Introduction

Let G = (V, E) be a graph consist of a finite non-empty set V of vertices and finite set E of edges such that each edge e_k is identified as an unordered pair of vertices $\{v_i, v_j\}$, where v_i, v_j are called end points of e_k . The edge e_k is also denoted by either $v_i v_j$ or $\overline{v_i v_j}$. We also write $G(V, E)$ for the graph. Vertex set and edge set of G are also denoted by $V(G)$ and $E(G)$ respectively. An edge associated with a vertex pair $\{v_i, v_i\}$ is called a self-loop. The number of edges associated with the vertex is the degree of the vertex, and d(v) denotes the degree of the vertex v. If there is more than one edge associated with a given pair of vertices, then these edges are called parallel edges or multiple edges. A graph that does not have self-loops or parallel edges is called a simple graph. We consider simple graphs only.

1.1 Definitions: (i) A graph G (V, E) is said to be a **star graph** if there exists a fixed vertex v (called the center of the star graph)such that E = {vu / u $\in$ V and u $\neq$ v}. A star graph is said to be an **n-star graph** if the number of vertices of the graph is n.
(ii) (Satyanarayana, Srinivasulu, and Mallikarjun [11]): The left zero divisor graph of R (denoted by LZDG(R)) is defined as follows: $V\left(LZDG\left(R\right)\right) = R = \left\{v_i / 0 \le i \le n\right\}$, and $E\left(LZDG\left(R\right)\right) = \left\{\overline{v_i v_j} / v_i v_j = 0 \text{ with } i < j\right\}$.
For other preliminary results and notations we use [18], [20] or [21]

Section 2: Star graph generated by x

2.1 Definition: Let G be a graph and $x \in V(G)$. A star graph generated by x is the subgraph S_x of G with $V(S_x) = \{y \in V(G) / \overline{xy} \in E(G)\}$ and $E(S_x) = \{\overline{xy} / y \in V(G) \text{ and } \overline{xy} \in E(G)\}$. In other words, the star graph generated by $x \in V(G)$ is the graph formed by the set of all vertices in $V(G)$ which are adjacent to x; and the set of all edges which are incident on x.

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2.2 Note: Suppose H is a n-star graph with centre x.
(i). Then (by the definition of n-star graph) there exist vertices $\{v_i / 1 \le i \le n-1\}$ in H such that $x \notin \{v_i / 1 \le i \le n-1\}$, $V(H) = \{x\} \cup \{v_i / 1 \le i \le n-1\}$ and $E(H) = \{\overline{xv_i} / 1 \le i \le n-1\}$.
(ii). Also $|V(H)| = n$.

2.3 Examples: Consider the graph G given by the Fig. 2.3.

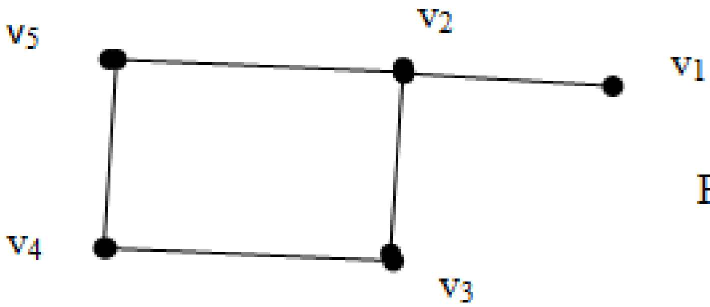

Fig. 2.3

(i) If $x = v_1$, then $V(S_x) = \{v_1, v_2\}$ and $E(S_x) = \{\overline{v_1v_2}\}$. So the star graph generated by v_1 is a 2-star graph.
(ii) If $x = v_4$, then $V(S_x) = \{v_2, v_3, v_4, v_5\}$, the set of all vertices adjacent to v_4 including v_4 $E(S_x) = \{\overline{v_2v_4}, \overline{v_3v_4}, \overline{v_5v_4}\}$, the set of all edges incident on v_4. So the star graph generated by v_4 is a 4-star graph.
(iii) If $x = v_2$, then $V(S_x) = V(G)$ and $E(S_x) = \{\overline{v_1v_2}, \overline{v_3v_2}, \overline{v_4v_2}, \overline{v_3v_2}\}$. So the star graph generated by v_2 is 5-star graph.

2.4 Result: Every star graph with centre x (which is a subgraph of G) is contained in the star graph S_x generated by x.
Proof : Let H be a star graph with centre 'x'. Then $\overline{xy} \in E(H)$ for every $y \in V(H) \setminus \{x\}$. Consider the subgraph S_x (the star graph generated by x). To show that H is a subgraph of S_x we show that $V(H) \subseteq V(S_x)$ and $E(H) \subseteq E(S_x)$. Let $u \in V(H)$. Then $\overline{ux} \in E(H) \subseteq E(G)$. So u is adjacent to x and so $u \in V(S_x)$ (by the definition of S_x).
Let $e \in E(H)$. Then $e = \overline{xy}$ for some $y \in V(H)$. Now the edge $\overline{xy}$ is incident on x and so $\overline{xy} \in E(S_x)$. Hence H is a subgraph of S_x. The proof is complete.

Section – 3: Star number of a graph
3.1 Definition: Let G be a graph. The star number of G is defined as
max {n / there exists an n-star graph which is a subgraph of G and n is an integer with $n \ge 1$}. We denote this star number of G by $S_n(G)$.

3.2 Result: If G is a graph, then $S_n(G) \le min\{|V(G)|, |E(G)| + 1\}$
Proof: Suppose there is an n–star graph H which is a subgraph of G. As we know n–star graph contains n vertices and so $|V(H)| = n$. Now n = $|V(H)| \le |V(G)|$ (Since H is a subgraph of G). Therefore $S_n(G) \le |V(G)|$. Since H is a tree, by Theorem 13.6 (p 372 of Satyanarayana and Syam Prasad [21]), we have that $|E(H)| = |E(V)| - 1 = n - 1$.
So $n - 1 = E(H) \le |E(G)|$ (since H is a subgraph of G). Thus $n \le |E(G)| + 1$.
Therefore we conclude that $S_n(G) \le |E(G)| + 1$
Hence we proved that $S_n(G) \le min\{|V(G)|, |E(G)| + 1\}$

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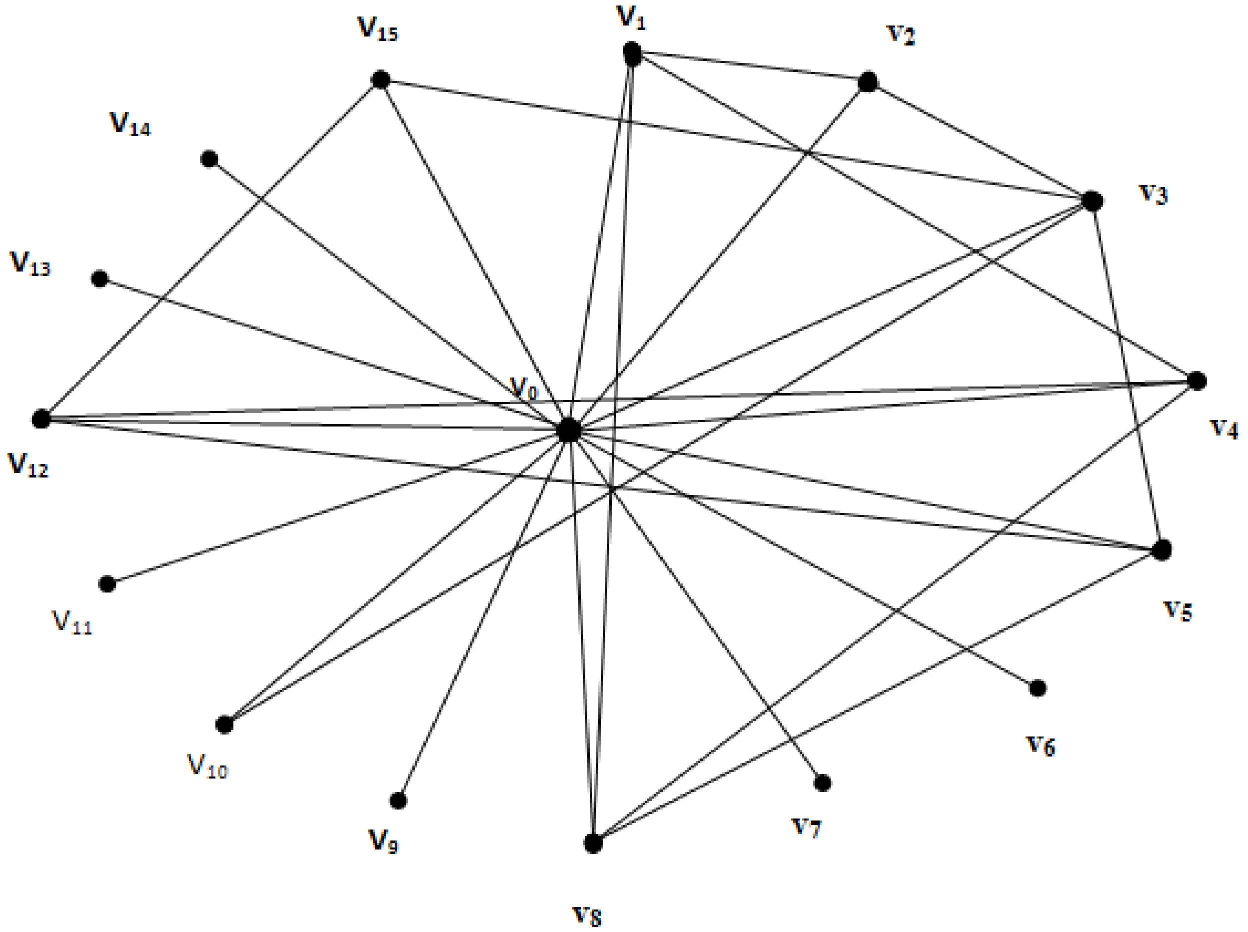
3.3 Example: Consider the graph given by the Fig.2.3.

By example 2.3(iii) G contains a subgraph S_{v_2} which is also 5-star graph. So $5 \leq S_n(G)$. By the Result 3.2, we have that $S_n(G) \leq |V(G)| = 5$. Therefore $S_n(G) = 5$. Thus the star number of the graph G is 5.

3.4 Theorem: The Star number of LZDG(R) where R= (2×2 matrix ring over Z_2) is 16.

Proof: The graph of LZDG(R) is given by Figure 3.3 (refer construction 2.4 on page 878 of Satyanarayana, Srinivasulu, and Mallikarjun [11])



Since $\{\overline{v_0 v_i} / 1 \leq i \leq 15\}$ is an n-star graph which is a subgraph of LZDG(R), we have that star number of LZDG(R) ≥ 16. In a contrary way, suppose that there exist an m-star graph H with centre x which is a subgraph of LZDG(R) and $m > 16$. By Result -1, H is a subgraph of S_x , where S_x is the star graph generated by the vertex x;

Now $x \in V(R) = \{v_i / 0 \leq i \leq 15\}$

If $x = v_0$ then by Note 2.5 page 879 of Satyanarayana, Srinivasulu and Mallikarjun [11] we have that $S_x = S_{v_0}$ and so m =16, a contradiction.

If $x = v_1$, then $16 < |V(S_x)| = |V(S_{v_1})| = 5$, a contradiction.

If $x = v_2$, then $16 < |V(S_x)| = |V(S_{v_2})| = 4$, a contradiction.

If $x = v_3$, then $16 < |V(S_x)| = |V(S_{v_3})| = 6$, a contradiction.

If $x = v_4$, then $16 < |V(S_x)| = |V(S_{v_4})| = 5$, a contradiction.

If $x = v_5$, then $16 < |V(S_x)| = |V(S_{v_5})| = 5$, a contradiction.
If $x = v_6$, then $16 < |V(S_x)| = |V(S_{v_6})| = 2$, a contradiction.
If $x = v_7$, then $16 < |V(S_x)| = |V(S_{v_7})| = 2$, a contradiction.
If $x = v_8$, then $16 < |V(S_x)| = |V(S_{v_8})| = 5$, a contradiction.
If $x = v_9$, then $16 < |V(S_x)| = |V(S_{v_9})| = 2$, a contradiction.
If $x = v_{10}$, then $16 < |V(S_x)| = |V(S_{v_{10}})| = 3$, a contradiction.
If $x = v_{11}$, then $16 < |V(S_x)| = |V(S_{v_{11}})| = 2$, a contradiction.
If $x = v_{12}$, then $16 < |V(S_x)| = |V(S_{v_{12}})| = 5$, a contradiction.
If $x = v_{13}$, then $16 < |V(S_x)| = |V(S_{v_{13}})| = 2$, a contradiction.
If $x = v_{14}$, then $16 < |V(S_x)| = |V(S_{v_{14}})| = 2$, a contradiction.
If $x = v_{15}$, then $16 < |V(S_x)| = |V(S_{v_{15}})| = 4$, a contradiction.
The proof is complete.

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