The International Journal Research Publication's
ISSN:2251 1563
Research Journal of Science & IT Management

# The Reliability Evaluation of Cube Based Interconnection Networks Under Node Failure Model

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**Abstract:** Under node failure model, a cube may operate in a gracefully degradable manner by supporting parallel algorithms in smaller fault-free cubes. In order to reduce execution slowdown in cube with a given faults, it is essential to identify the maximum healthy sub cubes (maximal incomplete sub cube) in the faulty cube. This paper proposes a new method to identify all the maximal incomplete sub cubes present in a faulty cube taking maximum fault tolerance level i.e. number of faulty nodes is equal to the system dimension. The procedure is a distributed one, as every healthy node next to a failed one performs the same procedure independently and concurrently. Then the reliability expression for the cube is derived. This method is well supported by an efficient algorithm which runs polynomially. The proposed method is found to be simple, general and efficient and thus is applicable to all the cube based topologies.
**Index terms:-**Cube, Maximal Incomplete sub cube, Discarded region, Reliability.

## 1. INTRODUCTION:

As parallel computer communication systems are very much popular and commercially widely used in real time applications, therefore considerable interest and increasing efforts have been made to develop such large communication systems. A major part of it is a parallel computer interconnection network, which is used to interconnect a large number of standalone processors. Therefore a wide variety of interconnection networks have been proposed like rectangular meshes, trees, shuffle exchange networks, omega networks and binary cubes[1]and[2].One of the widely used topology is the binary cubes, also known as the Boolean n-cubes. Due to attractive properties like regularity, symmetry, small diameter, strong connectivity, recursive construction and partition ability the n-cube topology has enjoyed the largest popularity. These properties lead to simple routing, support for wide application spectrum and fault tolerance for communication systems [3].

The n-dimensional cube is composed of $2^n$ nodes and has n- edges per node, n-bit binary addresses are assign to the nodes to the cubes in such a way that an edge or link connects two nodes if and only if their binary addresses differ by a single bit [3].This Inter connection network supports large numbers of resources with small diameters. But the major drawbacks of the cube networks are the numbers of communication ports and channels per processors is the same as the logarithm of the total numbers of processors in the system. Therefore the number of communication ports and channels per processors increases by increasing the total number of processors in the system. This drawback seems to be waived in the case of incomplete cubes, which shows the emulation performance as the n/w scales up in size [5] and [6].

The probability of fault in a larger system is given due importance. Whenever a fault arises, an n-cube may operate in a gracefully degradable manner due to the execution of parallel algorithms in smaller fault free sub cubes[6], which are comprises of healthy nodes. In order to maintain cube topology in the presence of faults, researchers have proposed addition of spare nodes thereby replacing the failed components with spares. This results in a much larger system than what is attained by any conventional reconfiguration scheme which identifies only complete sub cube [7]. Also fault tolerance can be achieved by reconfiguring the larger system to smaller sized system after the occurrence of fault [4]. Unlike a complete one, an incomplete cube can be of any arbitrary size, i.e. can be used to interconnect systems with any numbers of processors, making it possible to finish a given batch of jobs faster than it's complete counterpart alone by supporting simultaneous execution of multiple jobs of different sizes by assigning more nodes to execute the job cooperatively. Chen et. al.[9] determine sub cubes in a faulty hypercube. Similar research work can be found in literature [6],[8]and[10].Thus reconfiguring a faulty n-cube in to a maximal incomplete cube tends to lower potential performance degradation . This motivates our study to propose a simple, general and recursive method for finding all the incomplete sub cubes of a cube.

With the increase in size, the complexity of the interconnection network increases there by corresponding increase in computational power to maintain acceptable performance under reliable conditions [11] and [12]. For this the reliability prediction of the cube network is quite essential, to be used in critical applications [13],[14]and[15].

This paper proposes an efficient distributed procedure for locating or identifying all maximal incomplete sub cubes present in a faulty n-cube. The concept of discarded regions eliminates those nodes impossible to be part of any fault free sub cube containing the given node. There by forming the maximal incomplete sub cube. This method is illustrated through a 3-dimensinal hypercube. Then a generalized reliability expression for the maximal incomplete sub cube has been derived, which is supported by an effective algorithm.

## 2. BACKGROUND:

### 2.1 Hypercube

An *n*-dimensional Hypercube ($HC_n$) network of *N* processing elements (PEs) is defined by the following routing functions:

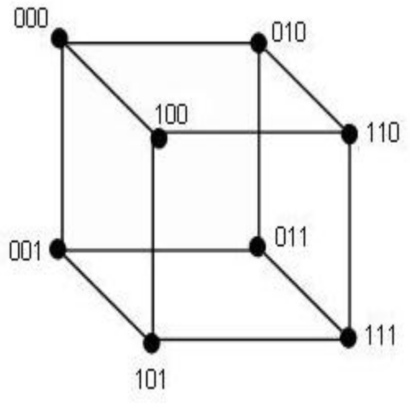
$$C_i(a_{n-1}\cdots a_1a_0)=a_{n-1}\cdots a_{i+1}\bar{a}_{i-1}\cdots a_1a_0\ ,\ \ for\ i=0,1,2,\cdots n-1 \qquad (2.1)$$

In the *n* cube, each processing element located at a corner is directly connected to *n* neighbors. The neighboring processing elements differ in exactly one bit position. There are $2^n$ number of processing elements and $n.2^{n-1}$ number of links in an *n*-dimensional hypercube ($HC_n$). A *3*-dimensional hypercube network topology is presented in Fig.2.1.

### 2.2 Folded Hypercube

As a variant of the hypercube, the *n* -dimensional folded hypercube ($FH_n$) is derived from the hypercube $HC_n$ by adding $2^{n-1}$ edges, called complementary edges. Each of them in between vertices, X= ($x_1, x_2$, ........, $x_n$) and $\bar{\bar{X}}=(\bar{x}_1,\bar{x}_2,\ldots\ldots\ldots\ldots\ldots\bar{x}_n)$ where $\bar{x}_i=1-x_i$. The $FH_3$ of dimension *3* is shown in Fig.2.2. It has been shown that $FH_3$ is (*n+1*) –regular (*n+1*)-connected. Like $HC_n$ , $FH_n$ is a Cayley graph and so $FH_n$ is vertex transitive.

The diameter of $FH_n$ is half of the diameter of $HC_n$. Thus the Folded hypercube is superior and is an enhanced version of $HC_n$. There are *n+1* internally disjoint paths of length at most *(n/2)+1*, between any pair of vertices in $FH_n$. The deletion of less than [*n/2*]-*2* vertices or edges does not increases the diameter of $FH_n$ and the deletion of up to *n* vertices or edges increase the diameter by at most one. The above properties mean that interconnection networks modeled by $FH_n$ are extremely robust.




n=3)

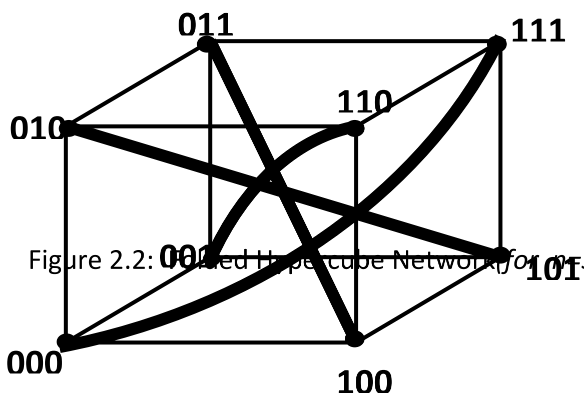



Figure 2.2: Folded Hypercube Network(for n=3)

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lt-tolerant hypercube ($FTH_n$) network is constructed from a standard $n$-dimensional hypercube by establishing spare links between each node and its farthest node. The spare links joining one node and its farthest node is called a complementary link or c-link. The number of c-links is $2^{n-1}$ and thus, the link redundancy is $1/n$ Nodes in the $FTH_n$ are assigned binary labels from $0$ to $2^{n-1}$ such that labels of two neighboring nodes differ in only one bit. Let $< x_0, x_1, \ldots, x_{n-1} >$ represent the dimension of $n$-cube. The $FTH_n$ can be visualized as a network with $n+1$ dimensions $< x_0, x_1, \ldots, x_{n-1}, x_n >$ , where $x_n$ is the dimension corresponding to c-links. Therefore, all links along dimension $x_i$, called collectively sheaf $i$, connect nodes whose labels differ in the $i^{th}$ bit ($for\ 0 \le i \le n-1$) or nodes whose labels are complements (for $i = n$). Links belonging to sheaf $i$ are labeled $i$ for $i<n$, and labeled $c$ when $i = n$. The Fig.2.3 shows a $3$-dimensional Fault-tolerant hypercube.

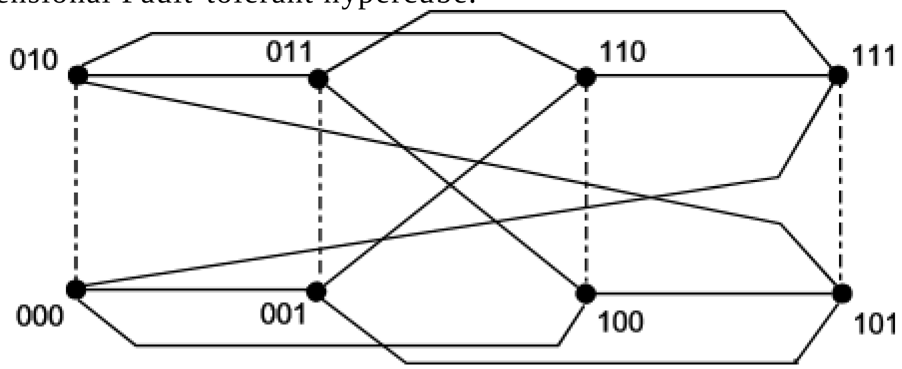



Figure 2.3: Fault-tolerant Hypercube Network *(for n=3)*

## 3. PROPOSED APPROACH FOR RELIABILITY EVALUATION:

The following notation and assumptions are made throughout this paper for reliability evaluation of cube-based interconnection networks.

### 3.1 Notation and Assumptions

*Notation:*

$IC_n$ *n-dimensional cube interconnection network*
$I_{n-1}$ *incomplete subcube*
$s$ *source node*
$d$ *destination node*
$\otimes$ *discarding operation*
$*$ *don't care symbol*
$n$ *system dimension*
$N$ *numbers of nodes in hypercube*
$u,v,w$ *adjacent nodes of source* node
$\bar{v}$, $\bar{w}$ *antipodal nodes of v, w*
$\lambda$ *node failure rate*
$t$ *mission time*
$G$ *probabilistic graph or Reliability Logic Graph*
$p_N$ *probability of success of node*
$q_N$ *probability of failure of node*

*Assumptions:*

1. Nodes failures are statistically independent.
2. No repair facility is available.

**3.2Reliability of Cube basedInterconnection Systems**

The failures of either the links or the nodes destroy the regularity of the cube interconnection topology. This leads to the formation of incomplete sub cubes. In this paper, a thorough analysis has been carried out for node failures independently.

Here, we assume all links to be perfect and nodes may be imperfect. For the purpose of analysis, the symbol $IC_n$ denotes an *n*-dimensional cube interconnection network. Each node in $IC_n$ is labeled by an *n*-bit string. For a given source node s, there exist a numbers of adjacent nodes, out of which at least one node is assumed to be faulty. Otherwise it will destroy the regularity property of $IC_n$. The addresses of the adjacent nodes differ by exactly one bit. Let us assume *u* to be the faulty node where as *v* and *w* are the non-faulty nodes. The nodes '*u*', '*v*', '*w*' are represented as binary strings. The $\bar{v}$ and $\overline{w}$ are the antipodal nodes of *v* and *w*. Taking bit operation $u \otimes \bar{v}$ and $u \otimes \overline{w}$ results *n* discarded regions. This leads to formation of an incomplete interconnection network $I_{n-1}^{m}$ where m is the numbers of nodes in fault free incomplete cube with dimension of *n-1*. Then the reliability of incomplete cube is evaluated by using the proposed algorithm.

*Definition 2.1:* Unlike complete cube an incomplete subcube can be constructed with any number of nodes to avoid the practical restriction of cube topology on the numbers of nodes in a system must be a power of *2*. A proper incomplete subcube in a faulty cube refers to a fault free incomplete subcube.

*Definition 2.2:* Each fault results in one such region known as discarded region which is the smallest subcube involving both the faulty and the antipodal nodes of adjacent *(n-1)* nodes. A discarded region is addressed by performing ⊗ operation on the labels of the faulty node and the antipodal node, where ⊗ is the bit operation defined as: it yields 0 (or 1) if the two corresponding bits are 0 (or 1), and it is ∗ if the two corresponding bits differ.

*Theorem 2.1*

For an *n*-dimensional cube with a given maximum tolerance level *n*, the network reliability can be expressed as:

$$R_{s-d} = {}^{n}C_1\, {}^{n-1}C_1\, {}^{n-2}C_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots {}^{2}C_1 \times {}^{2^{n-1}-2}C_{n-2} \times p^{n+1} \times (1-p)^{2^{n-1}-n}$$

*Proof:* Given a source node *s*, destination node *d* and the dimension of the graph as *n*, *Adj(s)= n*. Choosing a node as faulty it can be carried out in ${}^{n}C_1$ ways. Out of the remaining *n-1 Adj(s)* nodes the path from source to destination can be taken in ${}^{2^{n-1}-2}C_{n-2}$ ways.

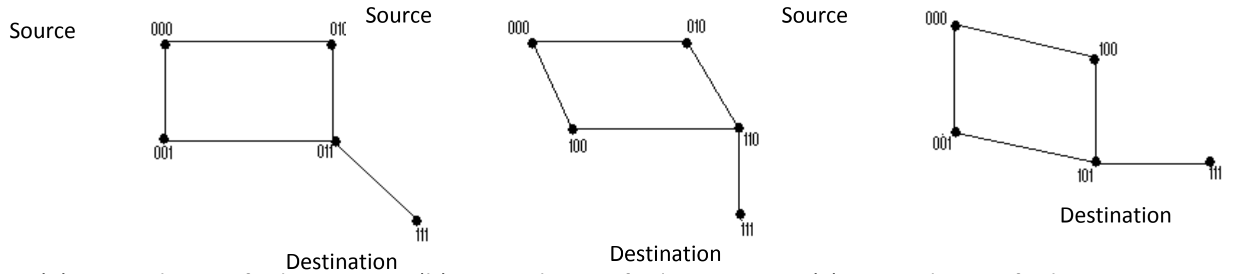
Now after discarding the faulty node from the cube network an incomplete subcube is obtained. If the given tolerance level is *n* i.e. *n* number of nodes can be faulty without disturbing a path from source *s* to destination *d*, then the total numbers of working nodes= $2^n - n$. Taking *p* asthe probability of success of a node, in the subcube$I_{n-1}$, choosing a node *n* from *Adj(s)* can be in ${}^{n-1}C_1$ i.e. out of *n-1* nodes one will work with probability *p*, which contributes the term *p*. Now since the source node and the destination node are fixed, one node $N \varepsilon I_{n-1}$ should be chosen to be working otherwise, the path will be destroyed. So without loss of generality the path contains *n* working nodes with probability *p* and $(2^{n-1}-n)$ numbers of failed nodes with probability *(1-p)*.

So the Reliability expression

$$R_{s-d} = {}^{n}C_1\, {}^{n-1}C_1\, {}^{n-2}C_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots {}^{2}C_1 \times {}^{2^{n-1}-2}C_{n-2} \times p^{n+1} \times (1-p)^{2^{n-1}-n} \qquad (3.1)$$

**3.3 Illustration**

Due to occurrence of faults because of node/link failures, a cube based system may result in incomplete subcubes. Therefore, it is essential to determine the subcubes for the purpose of reliability evaluation through a suitable method. The proposed method for determination of incomplete subcube is illustrated through a *3*-dimensional hypercube interconnection network.




(a): 100 taken as faulty (b): 001 taken as faulty (c): 010 taken as faulty

Figure 3.1: Incomplete sub cubes of a *3-D* hypercube

Consider the *3-D* Hypercube as shown in Fig.3.1 where the source node and destination node are labeled as 000 and 111 respectively. Out of the three adjacent nodes of source node, let the node 001 be faulty, then the antipodal nodes of the two other adjacent nodes are 011 and 101. Here a discarded region is addressed simply by performing operation ⊗ on the labels of the faulty node and the antipodal nodes. Let the faulty node is assumed to be 001, antipodal node of 100 is 011 and 001 ⊗ 011 = 0∗1 and 001 ⊗ 101 = ∗01. After removing these two discarded regions an incomplete subcube results which is shown in Fig.3.1 (b).The same operation can be performed by taking the nodes labeled 100 and 010 as faulty nodes. This results in two other incomplete subcubes as shown in Fig.3.1 (a) &3.1(c).

In the following subsection an efficient algorithm is proposed which generates the reliability expression R for the incomplete subcube in a recursive way. The proposed algorithm uses the node success probability $p_N$for evaluating the network reliability when nodes are assumed to be imperfect and links as perfect.

### 3.4 Proposed Algorithm

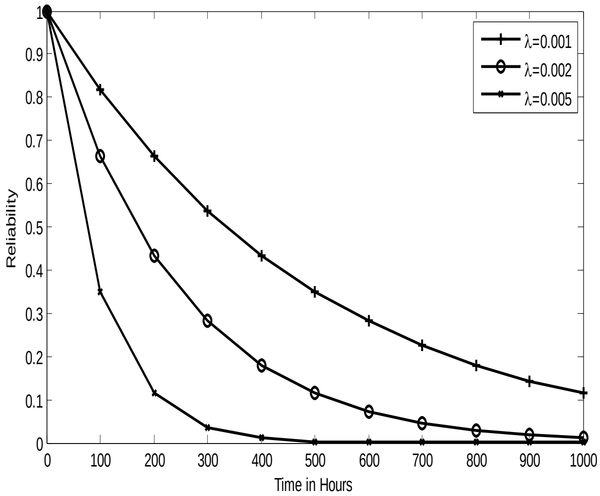
The algorithm is given below for reliability evaluation under fault situations.

*Reliability (G, s, d, n,* $p_N$ *,* $q_N$ *)*
{
*If* $(n \geq 2)$
{
*Adjacent=Adj(s)*
*Choose a node N from adjacent in* $^{n}C_1$ *ways.*
$N' = \{\text{Antipodal(N)}\}$
$V_i' = N' \otimes \{V \sim (s \cup d \cup N)\}$
$V' = \{V \sim (s \cup d \cup (Adj(s) \sim N)\}$
*fori=1 to* $|V'|$
*Discard* $N' \otimes V_i'$ *region*
$G' = (V', E')$
$R = R \times {}^{n}C_1 \; p^{n+1} q^{2^{n-1}-n}$
*Reliability (G, s, d, n,* $p_N$ *,* $q_N$ *)*
}
*else*
*return;*
}

## 4. RESULTS AND DISCUSSIONS:

The reliabilities of the three cube based interconnection networks have been evaluated by the proposed algorithm. The cube based interconnection networks those have been considered are: Hypercube ($HC_n$), Folded hypercube ($FH_n$) and Fault-tolerant Hypercube. The reliability of the individual networks are computed for different node failure rates $\lambda$ and plotted in Figs. 4.1-4.3.

In Fig.4.1, the network reliability values of $HC_n$ under different node failure rates are compared. Under low node failure rate such as $\lambda$=0.001, the reliability is observed to be 82% at mission time t=100 hours. However, the reliability of $HC_n$ drops down to 67% at a moderate node failure rate of $\lambda$=0.002 and it becomes less than 40% under high value of node failure rate i.e. $\lambda$=0.005.



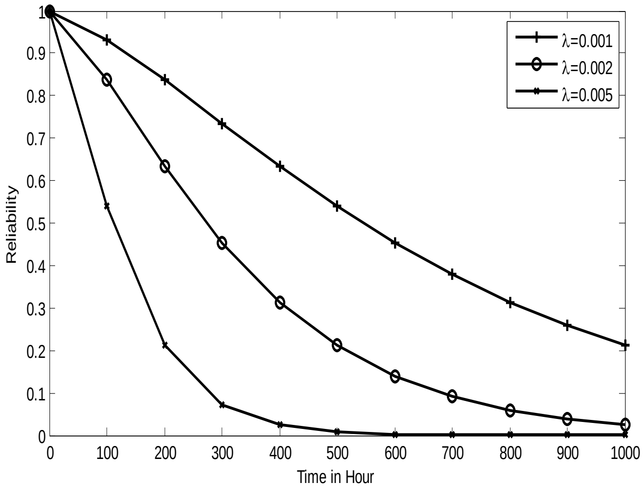




Fig. 4.1: Reliability of Hypercube ($HC_3$) Fig. 4.2: Reliability of Folded Hypercube ($FH_3$)

The reliability of the Foldedhypercube ($FH_n$) under different node failure rates is plotted in Fig. 4.2. The Folded hypercube provides a value of reliability as high as 95% under low node failure rate of 0.001 within 100 hours. The reliability falls down to 50% at 500 hours for the failure rate of 0.001. The reliability of $FH_n$ reaches to a value less than 3% at mission time below 400 hours under $\lambda$=0.002. It subsequently, becomes zero at mission time of 600 hours.

The Fault-tolerant hypercube ($FTH_n$) provides a reliability value which is slightly less than 90% at mission time 100 hours and $\lambda$ =0.001 (Fig.4.3). It degrades to 30% corresponding the following mission times and $\lambda$:

*t=700 hrs,* $\lambda$=0.001; *t=320 hrs,* $\lambda$=0.002; *t=150 hrs,* $\lambda$=0.005.

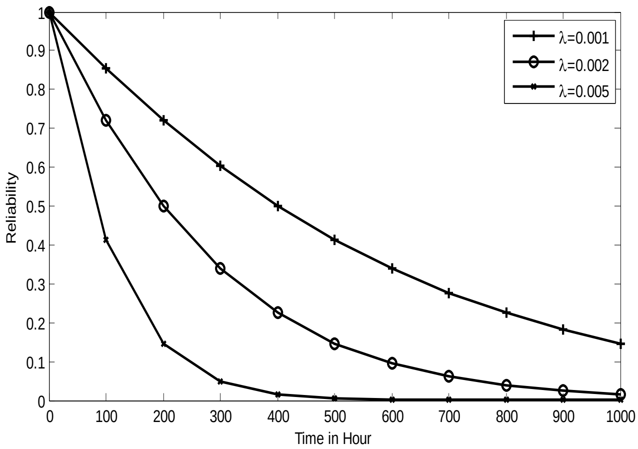



Fig. 4.3:Reliability of Fault-tolerant Hypercube ($FTH_3$)

The reliability of all the said three interconnection networks are compared on a common platform under the same node failure rate of $\lambda$=0.001. It can be observed that at mission time 100 hours, the reliability of Folded hypercube ($FH_n$) is found to be highest.

The reliability of Fault-tolerant hypercube ($FTH_n$) is next to Folded hypercube. The reliability of Folded hypercube ($FH_n$) and Fault-tolerant hypercube ($FTH_n$) are observed to be uniformly greater than that of the other network. The reliability of the said networks beyond the mission time 150 hours are found to be in the following order, where $n$ isthe dimension of the network.

$$FH_n > FTH_n > HC_n$$

## 5. CONCLUSION:

An efficient method has been proposed for identifying all fault free incomplete subcubes from a faulty cube by considering the maximum fault tolerance capacity. An efficient recursive algorithm is proposed to evaluate the reliability of the cube based topologies. While computing the reliability, imperfect nodes and perfect links are considered. The reliability of three important cube based networks viz. Hypercube, Folded hypercubeand Fault-tolerant hypercube are computed by the proposed method. From the computed results the $FH_n$ is found to be the best one in terms of its reliability among all the networks when the nodes are assumed imperfect. This method can be used for all types of cube based topologies which are operating in a gracefully degradable manner after occurrence of fault.

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