The application of queuing theory in the effective management of time in money deposit banks - A study of Zenith bank PLC in Enugu Metropolis

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ABSTRACT
Waiting for service is part of our daily life. We wait to eat in restaurants; we queue at the check-out counters in grocery stores, and we line up for service in post office. And the waiting phenomenon is not an experience limited to human beings only. Jobs wait to be processed on a machine, planes circle in stack before getting permission to land at an airport and cars stop at traffic lights. Waiting cannot be eliminated completely without incurring inordinate expenses and the goal is to reduce its adverse impact to tolerable levels. The objective of this study is to design a system such as the sum of costs of customers waiting and costs of idle facilities with special emphasis on Zenith Bank Plc in Enugu Metropolis. To achieve this major objective, data was collected by observation and queuing model was designed for the analysis. From the analysis, we discovered that the application of queuing theory can be used in the contest of a cost optimization model, where we seek the minimization of the sum of two costs: the cost of offering the service and the cost of waiting. We finally recommended that queuing theory is worth studying to enable the business executives determine and install the optimum service facilities so that the overall service cost is minimized.

INTRODUCTION
Generally speaking, a queue means a line up of people or objects. The queuing concept is built on the need to instill orderliness in human activities. The purpose of queuing is to receive attention from animate and inanimate objects. This attention is usually a service rendered by these objects which are commonly referred to as servers or service facilities. The main reason that necessitates Queuing is that the serving object usually has limited capacity to attend to needs required of them.

Queuing, is an old idea to man. But in Nigeria it was only in 1984 during the military Administration of General Mohammadu Buhari and late Tunde Idiagbon that great consciousness was created on the queuing concept (Egbo, 2001), That administration will always be remembered for the awareness created in Nigerians that the queuing concept can be used to improve human welfare. We are used to many queuing situations in life. Our forefathers applied the queuing idea to their problems. Labourers are known to queue at various farms who give them appointed days to help them in their farms. In economics, the scale of preference is a queue of human needs in their order of priority. In modern times, people queue in supermarkets to pay for goods collected, they queue in banks, filling stations, police check points and traffic points to receive service. People also queue at voter registration points and at polling booths to vote. People also queue at stadia and cinema halls to obtain tickets to watch a match or show. Students queue in schools for registration and other purposes. In the hospitals and medical clinics, patients queue to see the doctors and physicians. Jobs queue in machines at factories and production centers also at business centre and typing pools to receive attention. We also observe
that vehicles queue in mechanic workshops for routine maintenance and other purposes. In fact, the list of situations demanding queuing appears inexhaustible.

Today, businesses and other organizations adopt the queuing concept because of its positive impact on efficiency and economic use of resources. Service organizations employ queuing theory to reduce average time spent by a customer to receive service. This usually leads to customers’ satisfaction and increased patronage which automatically result to increased productivity. The arrival of customers creates opportunity for the delivery of goods and services which completes the process of production. Before the arrival of customers, it is expected that the instrument of service such as personnel, machines, goods etc. have been put in place. If these facilities are in place and customers arrive at a uniform rate, there may be no need for queues. Unfortunately, this is not always the case and so queues arise.

STATEMENT OF THE PROBLEM
The concept of providing and receiving services is inherent in today's specialized and interdependent world. Indeed, one of the most common phenomena of modern life is that customers such as human beings or physical entities, requiring service arrive at a set of service facilities that provides services. If upon arrival, the service facilities are free, customers are provided services without waiting. However, if the service facilities are not free, the customers either wait in a queue to receive their turn for service or they get discouraged by seeing the queue length and decide not to join the queue. This situation is often found in our money deposit banks and other service industries. It has been observed that in these outlets, our money deposit banks and other service industries, facilities have limited capacity to meet customers’ needs or demand; as a result, there are differences in the arrival of customers and the time taken to render services. Also, some services have peak periods or days. Such as days before any public holiday in our commercial banks when customers outnumber the facilities available. A breakdown of the service facilities such as machines (network failure), limited personnel and inefficient use of resources usually arise. These situations give rise to the formation of long queues which usually leads to customer's dissatisfaction, low patronage and eventual attrition.

Generally, a queuing or waiting-line problem arises whenever the demand for customer service cannot perfectly be matched by a set of well defined service facilities. The perfect match cannot be achieved because, in many situations, neither the arrival times or arrival rate of the customers nor the service times or service rate of the service facilities can be accurately predicted. This, in other words means that both the customer arrivals and service times are random. Consequently, either the customers have to wait for services (in this case queue emerges), or the service facilities have to wait (in this case. service facilities remain idle). Obviously, specific costs are associated with the waiting of the customers and the idle service facilities. According to Taha (1982), these two categories of costs move in opposite directions. For example, by adding more service facilities, we decrease the cost of customers waiting but increase the cost of idle facilities. Conversely, by decreasing the number of service facilities, we increase the cost of waiting but decrease the cost of idle facilities. The problem therefore arises on how to manage these costs for effectiveness and efficiency in the organization, hence this study.

OBJECTIVES OF THE STUDY
I. To investigate the applicability of queuing model in the management of time in Zenith Bank Plc.
II. To design a system that minimizes the cost of customers waiting and cost of idle facilities in Zenith Bank.
III. To examine and explain the operating characteristics of queuing system and provide equations for calculating their numerical values for effective decision making in Zenith Bank.
IV. To make recommendations based on the analysis.
REVIEW OF RELATED LITERATURE
This section titled Review of Related Literature dwells on discussing the views of other scholars as it relates to the topic under study. The study reviews the following sub topics: concept of queuing theory, basic structure and components of queuing system, types of queues, elements of a queue, and operating characteristics of a queuing system. These topics reviewed contributed to the conclusion and recommendation of the study.

THEORETICAL FRAMEWORK

CONCEPT OF QUEUING: According to Taylor, (1996) waiting in queues or waiting lines is one of the most common occurrences in every one's life. Anyone who has gone shopping or to a movie has experienced the inconvenience of waiting in line to make purchase or buy a ticket. Not only do people spend a significant portion of their time waiting in lines, but products queue up in production plants, machinery waits in line to be served, planes wait to take off and land, and so on. Since time is a valuable resource, the reduction of waiting time becomes an important topic of analysis. In their view, Anderson et al (1991), quantitative models have developed to help managers understand and make better decisions concerning the operation of waiting lines. In management science terminology, a waiting line is referred to as a queue, and the body of knowledge dealing with waiting line is known as queuing theory.
In the early 1905, A.K. Erlang, a Danish telephone engineer, began a study or the congestion and waiting times occurring in the completion of telephonic calls since then, queuing theory has grown far more sophisticated and has been applied to a wide variety of waiting his situations (Bose. 2001 ). In his view, Talia (2007), the study of queues deals with quantifying the phenomenon of waiting in lines using representative measure of performance, such as average queue length, average waiting time in queue, and average facility utilization. The result of queuing analysis can be used in the context of a cost optimization model, where we seek the minimization of the sum of two costs: the cost of offering the service and the cost of waiting. The main obstacle in implementing cost models is the difficulty of obtaining reliable estimates of the cost of waiting, particularly when human behaviour is an integral part of the operation.
According to Okeke (1995), queue is defined as a collection of objects awaiting service. It is usual to think of objects in the queue as human beings only. In studying queuing systems, we need to broaden our thought of the objects involved to include for example cars awaiting routine service, or stopping to buy fuel at a filling station, machines awaiting repairs, files, in fact any discrete object which needs one type of function or the other to be performed on it. Queues are in fact about delays. Whether the actual queue is observed or not, queues form because it is not at all impossible to organize the supplies to be exactly equal to the demand.

BASIC STRUCTURE AND COMPONENTS OF A QUEUING SYSTEM
In their view Dannenbring and Stair (1981), a queuing system essentially consists of the following four components;
   a. An input source or calling population that generates customer.
   b. A service system that consists of one of more service facilities.
   c. A queue that indicates the number of customers waiting for (the queue does not include customers being served). When the customers arrive at the service facilities, they examine the queue conditions and then decide whether or not to join the queue. Some customers are discouraged by the length of the queue and therefore do not join the line some customers after waiting in the queue for sometime become impatient and drop out of the queue.
   d. A queue discipline or service discipline according to which the customers are selected for service. The queue discipline indicates the decision rate for service. For example, in the campus refectories, students are usually served on a “first-come, first-served” basis. However, in a hospital emergency room, the service may be rendered on the basis of some medical priority.
INPUT SOURCE: The input source ur process generates customers for the service facilities. According to Talia (1982), the characteristics of the input source are given by:
1. The size of the calling population.
2. Arrival size.
3. Arrival control.
4. Arrival distribution.
5. The attitude of the customers.

SIZE OF THE CALLING POPULATION: the size of the calling population is **FINITE** if the number currently in the system (customers being served plus those in the queue) is a significant portion of the number of potential arrivals. For example input source of six cars that break down from time to time and require repairs. In this case, it is obvious that the number of actual car break down varies from one o six, and hence the number in the service system (i.e. the number of cars being repaired) is a significant portion of the number of potential arrivals. Hence, this is a case of finite population size. The population size is unlimited or infinite when the number currently in the system is an insignificant portion of the number of potential arrivals.

ARRIVAL SIZE: In his view, Taha (1982), an arrival can be single or batch.

ARRIVAL CONTROL: Most of the arrivals are subject to some degree of influence and control for example, cinema houses influence, to varying degrees the arrivals of their customers by charging discriminatory rates for films shown on weekends and weekdays. In his view Taha (1982), the queuing literature classifies control in the categories of controllable (i.e. registration days for college students) and uncontrollable (i.e. emergency room of a hospital)

ARRIVAL DISTRIBUTION (pattern of arrivals); According to Loomba (1978), the pattern of arrival is usually given by:
i. The distribution of times between successive arrival (i.e. inter arrival times) or
ii. The distribution of the number of arrivals per unit of time (i.e. arrival rate distribution).

The pattern or distribution of arrivals can be constant or random. A constant distribution means that the time intervals between arrivals (inter arrival times) are constant. Constant distributions are common in automated assembly line operations where parts, subassemblies, and finished goods arrive after predetermined time intervals.

A random distribution means that the inter arrival times cannot be predicted with certainty, and hence their pattern is given by actual empirical data or can be approximated by theoretical probability, such as the exponential distribution. The inter arrival times of many real-life phenomena (arrival at banks, gasoline station) can often be approximated by exponential distributions.

The distribution of the arrival rates (i.e. the number of arrivals per unit of time) of many real-life phenomena can often be approximately by the Poisson distribution.

The means of the Poisson and exponential distributions are inversely related as follows:

Poisson arrival rate
Mean arrival rate = \( \lambda \)

Exponential inter arrival Times
Mean inter arrival time = \( 1/\lambda \)

THE ATTITUDE OF CUSTOMERS: The attitude of customers is important because it affects the length of the queue. The attitude is a reflection of different types of customers such as;
i. Impatient customers (Balking or reneging),
ii. Patient customer (voluntary or involuntary).

When the customers arrive for service, they can either be immediately served (if the service facility is free) or they have to stand in the queue (if they wish to receive service). The customers who either do not join the queue (balking) or leave the queue before receiving service (reneging) are called impatient
customers. The customers who either voluntarily (e.g. patients in physician’s office) voluntary (eg. Prisoners or physical entities) remain in the queue until they are called patient customers (Bose, 2001). The purpose of recognizing the attitude of the customers is that, if a waiting time of extraordinary length produces a substantial number of impatient customers, then there is the potential of lost sales.

THE SERVICE SYSTEM: According to Lipsky (1992), the service system is characterized by the configuration (structure) of the service facilities and the service distribution. Configuration (structure) of the service facilities: Depending upon the nature of the service process, service facilities can be classified in terms of their configuration of channels (single or multiple) and phases (single or multiple). The term channel refers to the number of points of entry to the service system (Lipsky (1992). A single channel means that there is only one point of entry. Multiple channels refer to the parallel arrangement of service facilities i.e. Two or more points of entry exist so that two or more service stations can simultaneously begin the service process.

The term phase refers to the number of service stations through which the customer must pass before the service is considered complete. A single phase implies that there exists only one service station. Multiple phase refer to the service arrangement of service facilities that is, customer must go through two or more service stations in sequence before the service is considered complete (Lipsky, 1992).

Service distribution (Pattern of service): The pattern of service can be recorded by:

i. The distribution of service times.
ii. The distribution of the number of customers served per unit time (i.e. service rate).

As in the case of arrival, the distribution of service time can be constant or random. The constant service time occurs mostly in mechanized operations. In most real-life situations service times are random and can often be approximated by the exponential probability distribution. If the service is random but is specified in terms of service rates, it can often be approximated by the Poisson distribution (Loomba, 1978). Note that exponential service-time distribution gives rise to Poisson service rate (and vice -versa). The means of the two distributions are inversely related.

THE QUEUE: According to Morse (1958). The number of waiting lines and their respective lengths are the two basic aspects of the “Queue”. The number of waiting lines is essentially a function of the configuration of the service facilities. (That is, one waiting line for each different point of entry into the service system. Therefore, it could form a single queue or multiple queues. In his view Lee (1966), the length or size of the queue is influenced by such factors as;

- a. Physical space, (Example, limited space at a banking hall or gas stations).
- b. Legal restrictions example, city ordinance against forming queues on specified city streets.
- c. Attitude of the customers example, long lines discourage some customers from joining the queue.
- d. The relationship of the capacity of the input source to the capacity of the service facilities.

The length of the queue can be finite or infinite. The queue is finite (truncated) when there is a limit beyond which it cannot increase, example, the queue at a gas station). The queue is infinite when there is no limit on its size example, the number of mail orders for development of photos, (Lee, 1966).

THE QUEUE DISCIPLINE: Saaty (1983) views that the queue discipline indicates the decision rule by which the customers are selected from the queue for service. In most queuing systems, the queue discipline is the first-come first-served (FCFS) or first in, first out (FIFO). However, other types of queue disciplines can be provided on the basis of assigned priorities. Some examples in the category of assigned priorities are given by the following decision rules for selecting the customer;

I. Emergency first, example, in a hospital emergency room.
II. Reservation first, example, in a restaurant.
III. Shortest processing time first, example, in a job shop.
IV. Preemptive priority example, when service to one customer is interrupted to provide service to another.

Another type is the random queue discipline that reflects a service system not operated in a well-organized manner example, service in a higher store with several salesmen. The specification and analysis of the queue discipline is important because it affects the operating characteristics of the queuing system.

TYPES OF QUEUES
According to Egbo (2001), there are three main varieties of queuing situations commonly found in many places and circumstances around us. These include;
   i. Single queue with single service point.
   ii. Multiple queues with multiple service point.

Single queue with single service point: In this type of queue, only one line is formed and only one facility gives service. Typical examples are patients waiting to receive medical attention from a doctor in the doctor’s clinic, and students queuing at their Head of Department’s Office for registration.

Single Queue with several service points: In this type, the queue elements can go to any available service point and receive service. This is the system of queuing adopted recently and operational in the savings account and current account unit of most Banks in Enugu metropolis. Such as First Bank Plc, Enugu Main, and United Bank for Africa Plc, Enugu Main. It is also found in most petrol filling stations in this era of fuel scarcity.

Multiple queues with multiple service points: In this case according to Egbo, there are several queues and several service facilities. This type of queue reduces average time a customer or element in a queue spends, all things being equal examples of this type of queue are found at the out-patient department (OPD) of the University of Nigeria Teaching Hospital (UNTH), Enugu where patients with different ailments queue differently waiting for doctors in their specialist field.

ELEMENTS OF A QUEUE
According to Bose, (2001), the principal customers in a queuing situation are the customer and the server. Customers are generated from a source. On arrival at a service facility, they can start service immediately or wait in a queue if the facility is busy. When a facility completes a service, it automatically “pulls” a waiting customer, if any from the queue. If the queue is empty, the facility becomes idle until a new customer arrives.

From the standpoint of analyzing queues, the arrival of customers is represented by the inter arrival time between successive customers and the service is described by the service time per time customer. Generally, the inter arrival and service times can be probabilistic as in the operation of a post office, or deterministic as in the arrival of applicants for job interviews.

In his view Egbo (2007), the elements of a queue are;
   a) Arrivals
   b) Queue
   c) Service
   d) Exit/Departure.

ARRIVALS: This represents the people or jobs coming into the system to receive service. The arrival pattern of these objects affects the way of ordering queues. The arrival may be systematic or it may be random or systematic random. The arrival rate is a measure of the time an object takes to join a queue.

QUEUE: This represents the actual time spent in waiting, for service as the objects arrival there is the likelihood that they will wait or queue for their turn.

SERVICE: This represent receiving attention. The purpose of coming to the system is to receive service. The service rate describes the time spent in actually receiving the service attention. Like the arrival rate, the service rate may be random or systematic or both.
EXIT/DEPARTURE: After receiving service, the object will leave the system. Exiting from the system is also referred to as departure. This departure completes the cycle.

OPERATING CHARACTERISTIC OF A QUEUING SYSTEM
The behaviour of a queuing system is described by such variables as; arrival rate, waiting time in the queue, service time, idle time of service facilities, total time spent by a customer in the queuing system, and so on. The distributions of such variables need to be known, along with the numerical values of their standard deviations, and probability measures that the variables be less than or more than, certain specified numbers. For example what is the probability that the waiting time is five minutes or less? The operating characteristics of queuing system refer to the values (i.e. mean, standard deviation, etc) of different variables that are needed to either evaluate the performance of an existing queuing system or to design a new system, (Tanner, 1995).

Examples, the operating characteristics emerge as a result of interaction among the different components of queuing system. The numerical values of various operating characteristics of an existing queuing system can be calculated either analytically (that is by using mathematical equations relating to specific queuing models) or by simulation.

MODEL FORMULATION
The parameter that relates to probabilities in a single channel queue is;

1. The traffic intensity: This is the probability that an object arriving at the service points has to wait for sometime before being attended to. An object arriving for service has to wait for service if on arrival it finds one or more objects already at the service point. This probability denoted by ‘rho’ is given by the ratio of the arrival rate to the service rate, so that we can write
   Traffic intensity: \( \lambda / \mu \), which in obedience to the probability theory lies between zero and one.

2. The probability that there are at least ‘n’ items in the system is given by \( t^n \). It is necessary to observe that being in the system implies all the objects in the queue together with the object that is receiving service.

3. The probability of ‘n’ objects in the system is given by
   \[
   P_n = \left( \frac{\lambda}{\mu} \right)^n \left(1 - \frac{\lambda}{\mu} \right)
   \]
   We now give the system parameters that are concerned with numbers.

4. The average number of objects in the system is given by
   \[
   n_s = \frac{\lambda}{\mu - \lambda}
   \]

5. The average length of the queue, which is the same as the average number of object in the queue i.e. excluding the objects that is receiving attention is given by;
   \[
   n_q = \frac{\lambda^2}{\mu (\mu - \lambda)}
   \]

6. The average length of the queue excluding zero queuing is given by
   \[
   n-o = \frac{\mu}{\mu - \lambda}
   \]

7. The average number of time an object is in the system which is also known as the average system process time; (A.S.P.T) is given by
   \[
   A.S.P.T = \frac{1}{\mu - \lambda}
   \]

8. Finally, the average number of time units that an object stays in the queue i.e. the average queuing time is given by
   \[
   A.Q.T. = \frac{\lambda}{\mu (\mu - \lambda)}.
   \]
MULTIPLE CHANNEL QUEUE: In the single channel queue situation, there is only one service point, while in this case, more than one server is available. It will be assumed that these service points are approached through only one line (queue).

The customer at the head of the queue has the option of going to the server who is idle. Another vital assumption here is that the service times at the service points are negative exponential and that the simple queue situation also apply. If we denote the number of service points by ‘C’, then as in single channel queue, the following formulae apply in each case.

i. The probability of an object having to wait for service is given by;
\[
\left( \frac{f(\ell)}{c!} \right) x P_o \frac{C!}{(l-\ell)^2}
\]

ii. The average number of objects in the system is;
\[
N_{sc} = \left( \frac{f(\ell)}{c!} \right)^c x P_o + \frac{C!}{(l-\ell)^2}
\]

iii. The average number of object awaiting service i.e. in the queue is given by;
\[
n_{qc} = \left( \frac{f(\ell)}{c!} \right)^c x P_o \frac{C!}{(l-\ell)^2}
\]

iv. The average time an object is in system is;
\[
A.S.P.T. = \frac{f(\ell)}{c!} x P_o + \frac{1}{p} \frac{C!}{(l-\ell)^2 C \mu}
\]

v. Finally the average time an object stays waiting for service i.e. On the queue is given by;
\[
T.Q.C = \frac{f(\ell)}{C!} x P_o \frac{C!}{(l-\ell)^2 C \mu}
\]

Where in all cases = \frac{\lambda}{C \mu} and
\[
P_o = C! \frac{(1-\ell)}{(P e^\ell + C!(1-\ell)}} \sum_{N=0}^{C-1} \frac{1}{n!} (f e)^n
\]

ASSUMPTIONS

1. The size of the calling, population is infinite. This assumption implies that input sources are unlimited.
2. The arrival rate distribution is approximated by a Poisson distribution,
3. There is no balking. This assumption implies that arriving customers always join the queue,
4. There is no reneging. This assumption implies that customers stay in line until served (i.e. patient customers).
5. The queue discipline is first come, first serve (FCFS).
6. The permissible length of the queue is infinite.
7. The service time distribution is approximated by an exponential distribution.
8. The rate of service is greater than rate of arrivals (i.e., N > 2).
PRESENTATION AND ANALYSIS OF DATA

Table 1 SINGLE – CHANNEL QUEUE SITUATION IN ZENITH BANK PLC

<table>
<thead>
<tr>
<th>Condition</th>
<th>Bulk Teller</th>
<th>Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival time</td>
<td>0</td>
<td>10 minutes</td>
</tr>
<tr>
<td>Service time</td>
<td>5 minutes / customer</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>5 minutes / customer</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>

Source: Field survey 2015

We observed that customers’ arrival at the Zenith Bank located at independence Layout Enugu almost every ten (10) minutes to deposit money. We also observed that the bulk-teller on duty takes about five (5) minutes to attend to a customer. However, as a result of continuous coming of the customers, a queue was formed.

The objective of this study is to design a queuing system that will optimize the performance of the bank staff by determining the following:

1. The proportion of the time the bulk-teller spends attending to the customers, (denoted by ‘P’).
2. The traffic intensity denoted by ‘P’.
3. The average number of customers in the bank at any given time denoted by ‘n’
4. The average time a customer stays in the bank denoted by “A.S.P.T”.
5. The average number of time units that a customer stays in the queue (the average queuing time) denoted by “A.Q.T”.
6. The probability that there are at least three (3) customers in the queue, denoted by "Pₙ”.

DATA ANALYSIS

We first determine the average arrival and service rates i.e. λ and μ respectively. Thus; since one customer arrives every 10 minutes, and we know that there are 60 minutes in 1 hour.

\[ \lambda = \frac{60}{10} = 6 \]

Similarly, since the bulk teller takes 5 minutes to attend to a customer, then in one hour (60 minutes), he will attend to

\[ \mu = \frac{60}{5} = 12 \]

(1) The bank staff is fully in service when he is serving one or more customers.

Recall \( P₀ = \ell^n (1 - \ell) \) and

\[ P₀ + P₁ + P₂ + P₃ + \ldots \ldots + P₁ = 1 \text{ so that} \]
\[ P₁ + P₂ + P₃ + \ldots \ldots + P₁ = 1 - P₀ \]

Which is the probability of one or more customers in system, which is also proportion of time the bulk teller is busy? So that we can write

\[ P = 1 - P₀ \]
\[ P = 1 - \ell^n (1 - \ell) \]
\[ P = 1 - (6/12)^n (1 - 6/2) \]
\[ P = 1 - (1 - 6/12) \]
\[ P = \frac{1}{2} \text{ or 0.5 of his time.} \]

(2) The traffic intensity is given by

\[ \ell = \frac{\lambda}{\mu} \]
\[ = \frac{6/12}{12} \]
\[ = \frac{1}{2} \text{ or 0.5 idle time.} \]

(3) The average number of customers in the bank is given by
The average time a customer is in the bank is given by
\[
A.S.P.T. = \frac{1}{\mu - \lambda}
\]
\[
= \frac{1}{12 - 6}
\]
\[
= \frac{1}{6} \text{ hr or 10 minutes.}
\]

The average number of time units that a customer stays in the queue (the average queuing time) is given by
\[
A.Q.T. = \frac{1}{\mu (\mu - \lambda)}
\]
\[
= \frac{6}{12 (12-6)}
\]
\[
= \frac{6}{72}
\]
\[
A.Q.T. = 0.0833 \text{ hr or 5 minutes.}
\]

The probability that there are at least three (3) customers in the queue is given by
\[
P^n = \left[ \frac{\lambda}{\mu} \right]^n \left[ \frac{1 - \lambda}{\mu} \right]
\]
\[
= \left[ \frac{6}{12} \right]^3 \left[ \frac{1-6}{12} \right]
\]
\[
= (1/2)^3 (1-1/2)
\]
\[
P^n = 0.125 (0.5)
\]
\[
P^n = 0.0625
\]

Table 2: MULTIPLE CHANNEL QUEUE SITUATIONS IN ZENITH BANK PLC

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Casher 1</th>
<th>Casher 2</th>
<th>Casher 3</th>
<th>Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival rate</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50/hr</td>
</tr>
<tr>
<td>Service time</td>
<td>3 min./customer</td>
<td>3 min./customer</td>
<td>3 min./customer</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>3 min./customer</td>
<td>3 min./customer</td>
<td>3 min./customer</td>
<td>50/hr</td>
</tr>
</tbody>
</table>

Source: field survey 2015

We observed that in Zenith Bank plc sited at Okpara Avenue in Enugu, customers requiring to withdraw money from the bank arrive at the bank at an average rate of 50 customers per hour. We equally observed that the three cashiers that attend to these customers take an average of (3) three minutes each to serve each customer. These three (3) service points are approached through only one access.

This study is to design a system that will optimize the performance of the cashiers and reduce the time spent by the customers in the bank, by determining the following;

1. The probability that a customer arriving at the bank has to wait for sometime before receiving attention.
2. The average number of customers waiting to receive attention.
3. The average number of customers in this bank at any time whose purpose is to withdraw money.
4. What is the average system process time for this set up?
5. How long on the average would a customer stay on the line before being attended to?

**DATA ANALYSIS**

Observe that the arrival rate \( \lambda = 50 \) customers per hour.

Service time = 3 minutes per customer so that

Service rate = \( \frac{60}{3} \)

Service rate \( \mu = 20 \)

The number of service points is \( c = 3 \) cashier, so that

\[
\ell = \frac{\lambda}{c \mu} = \frac{50}{3 \times 20} = 0.83
\]

For the other calculations, observe that the formulae required are all function of \( P_o \), so that we first compute \( P_o \) given by

\[
P_o = C! \frac{(1-\ell)}{(\ell c)^c + c!(1-\ell)} \left[ \sum_{n=0}^{C-1} \frac{1}{n!} (\ell c)^n \right]
\]

\[
= \frac{3! (1-0.83)}{(0.83 \times 3)^3} + 3!(1-0.83) \left[ \frac{1}{0! (0.83 \times 3)^0} + \frac{1}{1!} (0.83 \times 3)^1 + \frac{1}{2!} (0.83 \times 3)^2 \right]
\]

\[
= \frac{0.02}{22.1601} = 0.046
\]

**a.** So that the probability that a customer at the bank has to wait for service is given by

\[
P = \frac{(\ell c)^c}{C! (1-\ell)} \times P_o
\]

\[
= \frac{(0.83 \times 3)^3}{3! (1-0.83)} \times 0.046
\]

\[
P = 0.696
\]

**b.** The average number of customers waiting to receive attention i.e. the number in the queue by

\[
n_{qc} = \frac{\ell (\ell c)^c}{c! (1-\ell)^2} \times P_o
\]

\[
n_{qc} = \frac{0.83 (0.83 \times 3)^3}{3! (1-0.83)^2} \times 0.046
\]

\[
n_{qc} = 12.8137 \times 0.046
\]

\[
n_{qc} = 0.1734 = 3.399 \text{ or about 4 customers.}
\]

**c.** The average number of customer in this bank wishing to withdraw money i.e. in the system is given by

\[
n_{sc} = \frac{\ell (\ell c)^c}{c! (1-\ell)} \times P_o + \ell \frac{c}{c! (1-\ell)}
\]
d. The average system process time i.e. the average time a customer is expected to be in the system is given by

\[
A.S.P.T_{sc} = \frac{C! (1-\ell)^2}{0.83 (0.83 \times 3)^3 \times 0.46 + 0.83 \times 3} x 0.046 + 2.49 \\
\frac{3! (1-0.83)^2}{0.1734} \\
= 5.88 \text{ or about 6 customers.}
\]

e. The average time a customer can wait on the line i.e. (in the queue) is given by

\[
T_{qc} = \frac{(\ell e)^C \times P_o + 1}{C! (1-\ell)^2 C \mu} \\
= \frac{0.83 \times 3}{3! (1-0.83)^2 \times 3(20)} \times 0.046 + 1 \\
= 0.068 \text{ hr or about 4 minutes}
\]

**SUMMARY OF FINDING**

From the single channel queue situation, we found out that for the bank staff to optimize his performance he has to be busy in the office 50% of the time. We also discovered that the bank staff is to be idle 50% of the time. It was discovered that an average of one customer must be found in the bank who came to deposit money. The average time a customer stays in the bank is 10 minutes. That is to say that the teller must not spend more than 10 minutes attending to one customer, otherwise the customer may get annoyed and go without making the deposit. We finally discovered that the probability that there are at least three (3) customers in the queue is 0.0625.

From the multiple channel queue situation, we discovered that the probability that a customer at the bank has to wait (or queue up) for service is 0.696. The analysis also indicated that the number in the queue is about four customers. This implies that if more than four are in the queue it will affect the general performance of the bank. The analysis also indicated that about 6 customers are in the bank waiting to make withdrawal. The model shows that the customers are expected not to be in the queue more than 7 minutes. Finally, the analysis indicated that the average time a customer can wait on the line before he or she leaves whether served or not is 4 minutes.

**DISCUSSION**

**Objective I:** To Investigate the Applicability of Queuing Model in the Management of Time in Zenith Bank.

Based on the analysis of this study and the theoretical frame work on queuing theory, queuing model is applicable in the management of time and cost in service industries such as commercial banks (Zenith Bank), hospitals, hotels, educational institutions, supermarkets, motor mechanic workshops, transport companies and in manufacturing firms where jobs are processed in sequence. However, it is observed from the study that most of these service industries including Zenith Bank in Enugu metropolis do not apply queuing model in reality. This could be as a result of ignorance of how to apply the model.
One of the major problems associated with the non application of queuing model by commercial banks is the long queue we always see in the banks today. The result of this objective is in line with the findings of Okonkwo (2009) in his study titled “An application of queuing theory to a transport company in Benin city, Edo State of Nigeria” published in Nigerian Journal of Engineering Management Volume 10. The major objective of the study was to find out the applicability of queuing theory in Transport Company. From his analysis using queuing model, it was found out that queuing model is applicable in transport companies.

**Objective II:** To Design A System that Minimizes the Cost of Customers Waiting and the Cost of idle Facilities.

The study of queues deals with quantifying the phenomena of waiting in lines using representative measures of performance such as average queue lengths, average time in queue, and average facility utilization. The following example demonstrates how these measures can be used to design a service facility. The principal actors in a queuing situation are the customer and the server. Customers are generated from a source. On arrival at a service facility, they can start service immediately or wait in a queue if the facility is busy. When a facility completes a service, it automatically “pulls” a waiting customer, if any, from the queue. If the queue is empty, the facility becomes idle until a new customer arrives. The problem associated with this situation, is the cost of customers waiting and the idle facilities. To minimize these costs, queuing model is designed as shown in model formation in the preceding chapter of this study. In designing the system, the components of a queuing system, queue discipline, queue characteristics and queue assumptions are applied as indicated in the literature review.

**Objective III:** To Examine and Explain the Operating Characteristics of Queuing System and Provide Equations For Calculating their Numerical Values for Effective Decision Making.

A convenient notation for summarizing the characteristics of the queuing situation in Zenith Bank Plc and the equations for calculating their numerical values is given by the following format: For single channel queue:

- **Traffic Intensity:** This is denoted by \( \rho (\ell) \)
- The average number of objects in the system is denoted by \( n_s \).
- The average number of object in the queue is denoted by \( n_q \).
- The average length of the queue is denoted by \( n-o \).
- The average system process is denoted by \( A.S.P.T \).
- The average queuing time is denoted by \( A.Q.T \).
- The equations for calculating these characteristics are as contain in the analysis.

**MULTIPLE CHANNEL QUEUE.**

1. The probability that customers wait for service is denoted by ‘P’.
2. The average number of customers waiting in the queue is denoted by \( n_qc \).
3. The average system process time is denoted by \( A.S.P.Tsc \).
4. The average time a customer can wait on the line is denoted by \( Tqc \).

The equations for calculating these equations are as contained in the analysis.

**CONCLUSION**

The main objective of this paper was to apply queuing theory in the management of time in commercial banks. The paper therefore aims at designing a system that will optimize a stated measure of performance such as the sum of costs of customers waiting and cost of idle facilities. The result of queuing analysis can be used in the context of a cost optimization model, where we seek the minimization of the sum of two costs; the costs of offering the service and the cost of waiting. The main obstacle in implementing cost models is the difficulties of obtaining reliable estimates of the cost of waiting, particularly when human behaviour is an integral part of the operation.
RECOMMENDATIONS
Sequel to the findings of this study, we recommend the following:
1. Management of any organization should know that when an object or a person is idle in the queue there is always some cost involved.
2. Management should also be aware that in trying to increase the service facilities, cost are also involved. The greater the service facility the quicker the queue will disappear and the service capacity stays idle.
In general, queuing theory is worth studying to enable the business executives determine and install the optimum service facilities so that the overall service cost is minimized.

REFERENCES