Simulation M/M/1 Queueing Model in Railway Reservation System

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ABSTRACT

This article deals with simulation M/M/1 queueing system and to fit the appropriate queueing model for the Ticket reservation system at Salem Town Railway Junction. Simulation Random model had been used for design, procedural analysis and the performance assessment. Arrival and departure time of all the customers data has been collected and analyzed statistically.

Key words: M/M/1 queueing model, ticket reservation system, Simulation Random techniques, arrival rate, service rate, traffic intensity.

I. INTRODUCTION

Indian Railway has made necessary arrangements to reserve tickets through internet to avoid inconvenience caused by waiting in the queue for reserving a ticket. Still many people reserve tickets in the counter for various reasons. Sometime a person has to wait long time in the queue because of inadequate number of reservation counters in the station. This article is focused on a system simulation queueing model to see whether the existing counter provide adequate service at a speedy rate to public or there is a need for few more counters. Shoemaker (1982) had reviewed this simulation in computer network and simulation II.

Kalasky (1995) has done the research for manufacturing system in Modeling and simulation in system modeling and computer simulation, this article is to fit a model for ticket reservation system in Salem Town Railway station. In order to avoid inconvenience caused by waiting in the queue, the authorities of Indian Railway has made necessary arrangements to reserve tickets through internet. Sometime a person has to wait long time in the queue because of inadequate number of reservation counters in the station.

Objectives of this study to analyze the performance of the existing system in terms of average waiting time of a customer in the system, average number of customers in the system, average number of customers in queue, and traffic intensity etc.,

SIMULATION QUEUEING TECHNIQUE

Simulation is a mathematical and computer modeling technique for replicating real-world problem situations. It is a modeling approach primarily used to analyze probabilistic problems. Simulation is popular because it can be applied to virtually any type of problem. It does not normally provide a solution; instead it provides information that is used to make a decision. Analysis of waiting lines can be done in two ways. They are:

1. Single-server queues (SINGLEQ)
2. Multiple-server queues (MULTIPLEQ).

In single server queue, there exists only one server and one queue. The customers in the system are served in the order of their arrival (often called FIFO: First in, First out). Queueing theory arises from the use of powerful mathematical analysis to theoretically describe production processes along with
statistical probabilistic techniques to account for varying dynamic patterns within stage of a productive process.

QUEUE DISCIPLINE

The most common queue discipline is the "first come, first served" (FCFS), or "first in, first out" (FIFO) rule under which the customers are serviced in the strict order of their arrivals.

M/M/1 QUEUEING SYSTEM

This queueing system deals with the process in which arrivals and departures (services) occur randomly over time. Arrivals can be considered as birth to the system since if the system is in state $E_n$, and an arrival occurs, the state is changed to $E_{n+1}$. On the other hand a departure occurring while the system is in state $E_n$, sends the system down one to $E_{n-1}$, and can be looked upon as a death. This type of process is generally referred to as birth-death process.

II. POISSON PROCESS

The Poisson distribution may also be obtained independently. Let $x$, be the number of arrivals in time interval $t$ on customer arrivals. Consider the following experimental conditions.

1. The probability of getting service is a positive constant $\lambda$ and denotes a small increment in time $dt$.

2. The probability of getting more than one arrival in this time interval is very small that is, the order of $(dt)^2$, that is 0.

$$\lim \frac{(dt)^2}{(dt)} = 0$$

3. The probability of any particular service in the time interval $(t, t+dt)$ is independent of the actual time $t$ and also of all previous service times.

Under these conditions it can be shown that the probability of getting $x$ calls in time $t$ say $P_{(s)} = t$

$$P_{(s)}t = \frac{e^{-\lambda t}}{x}; x = 0, 1, 2, 3, \ldots, \infty$$

which is a Poisson distribution with parameter $\lambda t$

CHARACTERISTICS OF (M/M/1): (FIFO/$\infty$) QUEUEING SYSTEM

For this model, $\lambda$ is the arrival rate, $\mu$ is the service rate and traffic intensity is $\rho = \frac{\lambda}{\mu}$

Finding $P_0$ and $P_n$

Step 1. $P_1 = \frac{\lambda^2}{\mu} P_0, P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0, \ldots, P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$

Step 2. $\sum_{n=0}^{\infty} P_1 = 1, \text{then} P_0 \sum_{n=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^n = 1, \Rightarrow P_0 = \frac{1}{\sum_{n=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^n}$
Step 3. $\rho = \frac{\lambda}{\mu}$ then
\[ \sum_{n=0}^{\infty} \left( \frac{\lambda}{\mu} \right)^n = \sum_{n=0}^{\infty} \rho^n = \frac{1-\rho^n}{1-\rho} = \frac{1}{1-\rho} \{\rho < 1\} \]

Step 4. $P_0 = \frac{1}{\sum_{n=0}^{\infty} \rho^n} = 1-\rho$ and $P_n = \rho^n (1-\rho)$

1) Average number of customer in the system is given by
\[ L_s = \sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{\infty} n\rho^n (1-\rho) \]
\[ = (1-\rho) \rho \sum_{n=0}^{\infty} n\rho^{n-1} (1-\rho) \rho \frac{d}{d\rho} \left( \sum_{n=0}^{\infty} \rho^n \right) \]
\[ = (1-\rho) \rho \frac{d}{d\rho} \left( \frac{1}{1-\rho} \right) (1-\rho) \rho \left( \frac{1}{(1-\rho)^2} \right) \]
\[ \rho = \frac{\lambda}{1-\rho} \mu - \lambda \]

2) Average queue length $L_q$ is
\[ L_q = \lambda W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\lambda^2}{\mu(\mu-\lambda)} \]

3) Average waiting time of a customer $W_q$ is
\[ W_q = \frac{L_q}{\lambda} \quad \text{(or)} \quad \frac{\lambda}{\mu-\lambda} \]

4) Average waiting time of a customer $W_s$ in the system
\[ W_s = \frac{1}{\mu-\lambda} \]

The primary data collected for one day in the reservation counter which is shown in the Table 1.

<table>
<thead>
<tr>
<th>Arrival time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>13</td>
<td>25</td>
<td>37</td>
<td>45</td>
<td>52</td>
<td>30</td>
<td>26</td>
<td>17</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

It is observed that during the survey, totally 260 persons have utilized that services of counter. In order to check if the arrival pattern follows a Poisson distribution, the Poisson probabilities are calculated and chi-square test is applied.

Ho: The Poisson distribution is appropriate for the arrival pattern.
Table 2: Arrival of customers at reservation counter.

<table>
<thead>
<tr>
<th>Arrival Time in</th>
<th>Total no. of customers arrived</th>
<th>(fx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
<td>111</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>180</td>
</tr>
<tr>
<td>5</td>
<td>52</td>
<td>260</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>180</td>
</tr>
<tr>
<td>7</td>
<td>26</td>
<td>182</td>
</tr>
<tr>
<td>8</td>
<td>17</td>
<td>136</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>260</td>
<td>1252</td>
</tr>
</tbody>
</table>

The average inter-arrival time of customers is computed as
\[
\lambda = \bar{x} = \frac{\sum fx}{N} = \frac{1252}{260} = 4.82
\]

The Poisson distribution is fitted by taking \( \lambda = 4.82 \). The Poisson expected frequencies are computed and chi-square test applied by taking the null hypothesis.

The Chi-square values are calculated using the formula
\[
\chi^2 = \frac{(f_e - f_o)^2}{f_e} \sim \chi^2 \text{ with n-1 degrees of freedom.}
\]

Table 3: Fitting Poisson distribution

<table>
<thead>
<tr>
<th>Arrival Time in</th>
<th>Observation frequency</th>
<th>Expected frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>10.11</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>24.50</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
<td>39.31</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>47.36</td>
</tr>
<tr>
<td>5</td>
<td>52</td>
<td>45.62</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>36.67</td>
</tr>
<tr>
<td>7</td>
<td>26</td>
<td>25.25</td>
</tr>
<tr>
<td>8</td>
<td>17</td>
<td>15.22</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>8.15</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>3.93</td>
</tr>
</tbody>
</table>

The chi-square value is calculated to be 4.1314 and the corresponding table value is \( \chi^2_{(0.05,8)} = 15.507 \). Hence the calculated value is less than the Chi-square table value. So the null hypothesis accepted. It is concluded that the arrival pattern follows Poisson law.

The average arrival rate and the service rate are computed from the simulation random technique, which is shown below
\[
\lambda = 0.2083/\text{minutes} \quad \mu = 0.2381/\text{minutes}
\]

Since Poisson distribution is found to fit the arrival pattern and exponential distribution for service pattern, the queueing model for the existing system is taken as (M/M/1): (\( \infty / \text{FIFO} \)).

RESULTS
Average arrival rate $\lambda = 0.2083$
Service rate $\mu = 0.2381$

1) Average number of customer in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = 6.9899$$

2) Average queue length

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = 6.1151$$

3) Average waiting time of a customer

$$W_q = \frac{L_q}{\lambda}$$

$$\therefore W_q = 29.3572$$

4) Average waiting time of a customer $W_s$ in the system

$$W_s = \frac{1}{\mu - \lambda}$$

$$\therefore W_s = 33.5570$$

5) Traffic intensity $\rho = \frac{\lambda}{\mu} = 0.8748$

CONCLUSIONS

From the results the average waiting time of a customer in the queue 29.35 and the traffic intensity is 87%. From this we conclude that the system is almost busy. As the waiting time is more, multiserver facility can be implemented to minimize the waiting time of the customers.

REFERENCES