Back-testing Value at Risk models on Indian stock markets using Covariance and Historical Simulation approach

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Introduction:

Value at Risk (VaR) is the estimate of the potential decrease in the value of the security for the given time horizon and particularly captures the risk element arising out of fluctuations in the market. VaR is globally used as the risk measurement tool by the Banking sector, Investment sector and various other research analysts. Risk management for the banking sector has become crucial and closely monitored since the fall-out due to collapse of several global financial institutions like Lehman Brothers, Orange county etc. Basel Committee on Banking Supervision’s Basel norms emphasise this aspect of VaR under Capital Adequacy norms. There are various models to calculate Measure risk under VaR which can be used by not only Banking sector but also other sectors to enable de-risking of their business through predictive back-testing. Currently regulators do not mandate use of any specific model and hence banking sector is free to use any model.

This aim of the study is to back test two such models namely Variance Approach and Historical Simulation on the large data sample of 431 S&P company share prices on the BSE exchange and check the accuracy of these models in the Indian context. The same is examined on the portfolios of share each containing 10 companies shares on the predefined weight.

Keywords: Value at Risk, Covariance-Variance Approach, Historical Simulation, Back-testing, Markowitz’s theory.

Reviews of Literature:
Matthew Pritsker,(1996), in the paper titled “Evaluating Value at Risk Methodologies: Accuracy versus Computational Time” examined six VaR models and compared their computational time requirement and accuracy and also focused on the errors in the VaR models occurring due to the estimation of nonlinearity. The paper further provides insights on newer methods of using order statistics to create confidence levels interval for errors and errors as a true percent of Value at Risk for each methods.
Dai Bo, (2001), in the study titled “Value at Risk” explained in depth concept of VaR and measurement of the same under multiple parameters. Study further elaborates on Different mathematical and statistical tools examined which can enable efficient risk management, including marginal VaR, incremental VaR and component VaR. Apart from understanding the impact of time-varying using GARCH process, few back-testing models have been introduced.
Elizabeth Sheedy, (2008), in the paper titled “Why VaR models fail and What can be done” examines why VaR models that did not factor volatility clustering for market risk failed at many major financial institutions. While on one hand GARCH based solution is suggested to overcome this problem, objections voiced by experts in relation to GARCH-based risk measures have been analyzed to arrive at possible solutions to it, on the other hand, problem associated with implementation of Garch-based VaR model implementation has also been worked upon.
Carol Alexander, Jose María Sarabia, (2011), in the research paper titled “Value- At-Risk Model Risk” introduced a ‘top-down’ approach to compute additional market risk capital which has been
mandated due to amendment in banking regulations, on the existing firm wide VaR models used by large banks, by constructing a rigorous statistical framework to derive the add-on value adjustments which are easy to implement.  

Dario Brandolini, Stefano Colucci, (2012), in the paper titled “Back-testing value-at-risk: a comparison between filtered bootstrap and historical simulation” compared Value at Risk produced by two risk models: historical simulation and Monte Carlo simulation. Tests were conducted on three parameters viz unconditional coverage, independence and conditional coverage keeping VaR at 95% and confidence level was analyzed for one day horizon: S&P 500, Topix, Dax, MSCI United Kingdom, MSCI France, Italy Comit Globale, MSCI Canada, MSCI Emerging Markets.  

Dr. Anirban Ghatak, (2013), in the paper titled “Analysis of the Performance of VAR Models as a tool for Market Risk” examined the equally weighted moving average, exponentially weighted moving average and historical simulation approach on the portfolio and compared the impact of portfolio on these models.

Research Methodology:

**Problem statement:** There are various methods of calculating VaR and it is difficult to decipher as to which model is appropriate in the Indian context and hence this study focuses on the back-testing of variance approach and historical simulation approach on 431 S&P BSE companies share prices and verify it to get insights into robustness and accuracy of various models.

**Objective of the study:**  
- To understand the working of VaR models in estimating risk in securities for the one day time horizon.  
- To verify the accuracy of these models in Indian stock Market context.  
- To validate the efficacy of these models at different confidence levels.  
- To compare the effectiveness of these models for single scrips and for portfolio risk estimations.

**Data collection:** Historical share prices (daily closing on BSE exchange) of 431 S&P BSE companies (mentioned in Annexure 1) was collected from April 1st 2006 to March 31st 2012 for the estimation of VaR for April 1st 2012 from Prowess database. The VaR for each day was calculated on the rollover basis and checked with the next day fluctuation in share prices. The back-testing was done for 312 trading days i.e. from 1st April to 28th June 2013.

**Sample size and techniques**  
431 companies from BSE was used as sample and for each company observations for 313 trading days was captured and analysed at 95% and 99% confidence levels. The VaR estimates calculated at 95% confidence level were tested for accuracy at 95% confidence level and VaR estimates calculated at 99% confidence level were tested for accuracy at 99% confidence level.

**Variance approach:** Variance approach assumes that the daily returns on the assets (shares) follow normal distribution. Hence in order to estimate the VaR in single asset the normal distribution can be used as follows:  
VAR (1day) = Standard deviation * multiplication factor at given confidence level(z value)  
The multiplication factor depends on the confidence level

**Historical simulation:** Historical simulation assumes that the future is adequately represented by the history. It does not assume that the historical returns are normally distributed. Hence can be directly calculated using cumulative frequency distribution.  
VAR = percentile function of daily returns at required confidence level.
**Variance –covariance approach:** The VaR of the portfolio was calculated using Markowitz’s equation as follows:

\[ \text{VaR}(p) = \sqrt{\sum (\sigma_i^2 \cdot \sigma_j^2) + 2 \sum \sigma_i \sigma_j \cdot \rho_{ij} \cdot \sigma_i \sigma_j \cdot \rho_{ij} \cdot \sigma_i \sigma_j} \]

Where i and j are the set of pair of the assets in the portfolio, P is proportion of the ith asset, \( \rho \) represents the correlation factor between two shares and \( \sigma \) represents the standard deviation of the assets.

**Historical simulation for portfolio:** The VaR of the portfolio was calculated in similar fashion as that of single assets historical simulation. Here historical price changes are taken for the portfolio instead of the single asset.

The value change in the portfolio was calculated as

\[ \text{Returns of portfolio} = \sum w(i) \cdot p(i) \]

Where w represents weight of ith asset and p represents percentage change in ith asset.

**Back-testing:** back-testing is the process of checking the mathematical soundness of the VaR model to validate that the model has been implemented correctly. The VaR of each asset or portfolio of asset is calculated for each day and checked with the actual fluctuation of price of the asset/portfolio. If the loss in asset/porfolio is higher than estimated, then it generates an exception. The same process is carried out for the number of observations for the observation period.

**Exception test:** The model is accepted when it follows the following criteria

Number of exceptions = \( \alpha^* (\sqrt{(t \cdot (1-c)}) + t \cdot (1-c) \]

Where \( \alpha \) represents number of z value at given confidence level, t total number of observations and c is confidence level.

In order to test for the accuracy of the model POF test was done using Bernoulli equations. This means that at 99% confidence level the model works if the exception generated in asset is 7 or less and at 95% confidence level the exception limit being 21.

**Hypothesis to be tested:**

Hypothesis-1
Ho: Variance approach works well for single scrips at 95% confidence level.
H1: variance approach does not work for single scrips at 95% confidence level.

Hypothesis-2
Ho: historical simulation works well for single scrips at 95% confidence level.
H1: historical simulation does not work well for single scrips at 95% confidence level.

Hypothesis-3
Ho: Variance approach works well for single scrips at 99% confidence level.
H1: variance approach does not work for single scrips at 99% confidence level.

Hypothesis-4
Ho: historical simulation works well for single scrips at 99% confidence level.
H1: historical simulation does not work well for single scrips at 99% confidence level.

Hypothesis-5
Ho: variance approach works well for portfolio of scrips at 95% confidence level.
H1: variance approach does not work well for portfolio of scrips at 95% confidence level.

Hypothesis-6
Ho: historical simulation works well for portfolio of scrips at 95% confidence level.
H1: historical simulation does not work well for portfolio of scrips at 95% confidence level.
Hypothesis-7
Ho: variance approach works well for portfolio of scrips at 99% confidence level.
H1: variance approach does not work well for portfolio of scrips at 99% confidence level.

Hypothesis-8
Ho: historical simulation works well for portfolio of scrips at 99% confidence level.
H1: historical simulation does not work well for portfolio of scrips at 99% confidence level.

Observations and findings of testing of hypothesis related to single scrips:
Covariance approach( single scrips at 95% confidence level):
It was observed that all the 431 companies shares passed the test with variance approach (calculated at 95% confidence level) i.e. none of the selected company shares generated exceptions above 21 in the 312 days of back-testing period. Hence null hypothesis-1 stands true.

Historical simulation (single scrips at 95% confidence level):
It was observed that only seven companies shares viz AstraZeneca Pharma India Ltd, Elder Pharmaceuticals Ltd, G T L Ltd, Linde India Ltd, Manappuram Finance Ltd, Opto Circuits (India) Ltd and Tulip Telecom Ltd generated exception above the limit, hence failed in back testing process and the balance 423 companies shares passed the back-testing process for the model. Hence statistically null hypothesis-2 can be proven true.

Covariance approach( single scrips at 99% confidence level):
It was observed that except four companies shares all the other observed companies shares were within the exception limit of seven exceptions in the back-testing period. The companies which did not pass the test, are AstraZeneca Pharma India Ltd, Elder Pharmaceuticals Ltd, Manappuram Finance Ltd, Opto Circuits (India) Ltd. Hence at 99% percentile level, null hypothesis-3 stands true.

Historical simulation (single scrips at 99% confidence level):
Except Manappuram Finance Ltd. all the other 430 companies’ shares passed the back-testing process. The number of exceptions generated by Manappuram Finance Ltd. was 9 in the observation period. Hence fourth null hypothesis-4 stands true.
The number of exceptions generated in the single scrips predictive models were within the permissible level for significant number of companies hence both the model works well at 99% and 95% confidence levels.

Observations and findings of testing of hypothesis related to Portfolios:
Five portfolio each containing the 10 different companies shares (as mentioned in the annexure 2) was selected as the sample set for the research. Equal weights of each company shares were assumed in the portfolios. Back-testing for the same observation period of 312 days as taken in the single scrips was observed.
Fig 1. Back-testing of portfolio 1
Portfolio 1 did not generate any exceptions at 99% confidence level in both the models whereas at the 95% confidence level the historical simulation generated 3 exceptions and covariance approach showed no exceptions in the back-testing period (Fig 1).

Fig 2. Backtesting of portfolio 2
In portfolio 2 the number of exceptions generated at the 95% confidence level was 3 for both the models and no exceptions were observed at the 99% confidence level (Fig 2).

Fig 3. Back-testing of portfolio 3
Portfolio 3 showed no exceptions at the 99% and 95% confidence level for both the models (Fig 3).
Fig 4. Back-testing of portfolio 4
In portfolio 4 there were not any exceptions at 99% confidence level. But at the 95% confidence level, covariance approach had 3 exceptions and historical simulations had 4 exceptions in the observation period (fig 4).

Fig 5. Back-testing of portfolio 5
Portfolio 5 did not have any exceptions at 99% confidence level, whereas at 95% confidence level VaR, covariance approach generated 2 exceptions and historical simulation generated 3 exceptions in the observation period.

The number of exceptions generated by both models at 95% and 99% confidence level were within the permissible level of significance i.e. 7 at 99% confidence level and 21 at 95% confidence level). Hence all null from hypothesis-5 to hypothesis-8 for portfolio stands true.

Findings:

1. At the given confidence level VaR calculated using different methods provided the significant risk measure for single as well as portfolio of assets. Although it was found that number of exceptions varied according to the model, both the model as a whole can be used as a risk measurement tool.

2. It was clear from the observations that historical simulation generated far less exception at the 99% confidence level than covariance approach in single scrips. The returns of many scrips had fatter tails as compared to what normal distributions predicts. Hence VaR predicted by the covariance approach at 99% confidence level was below what was actually observed in the historical simulation method.
3. On the other hand at 95% confidence level the number of exceptions generated in the covariance approach are less compared to historical simulation. Hence at 95% confidence level, covariance approach works as a better risk measurement tool in single scrips.

4. It was observed that both the models could predict the risk in portfolio of assets much better than that in the single assets. Hence the prediction of portfolio of asset was more accurate with these models than single assets.

Conclusions: The results from this research prove that both historical simulation and covariance approach effectively predict the VaR in the sample of 431 S&P BSE companies’ stock prices. These companies comprised of various sectors like Banking services, computer and IT services, automobiles, Energy, Cement, FMCG, Tires and many others. Hence with this diversified number of sample companies, it can be inferred that VaR in the Indian stock market can be predicted by these models and the same can be applied to other companies across sectors not considered in the sample set herein. The confidence level to be used in the estimating VaR depends on the risk appetite of the company and market conditions. It is preferred to use higher confidence levels during economic crisis.

Suggestions: Investment is about managing risk and returns. Financial institutions face high amount of risk in their investments, but find it difficult to accurately quantify it. Increasing instability in the financial sector has compelled organizations to develop various models for risk management and update it from time to time. Basel committee has allowed organizations to build their own VaR models and manage it. This study proves that Covariance approach and historical simulation method of estimating VaR are effective in the Indian Stock market context. This method can be used by investment managers for estimating risk in the portfolio, individuals for their investments, bankers for estimating market risk as per the Basel norms and other financial institutions. Back-testing of these models on a regular basis is required in order to validate the soundness of the models and to take decisions based on the prevailing market conditions. This research focused on the 1 day time horizon of securities for evaluations, but the time horizon can be altered according to the requirement of the organization for the same models.

References:


