On Fuzzy Modeling in Management

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Abstract
In the process of solving the control tasks of complex objects often have to deal with the uncertainty of the environment functioning. For example, in economics and management have to make decisions in an uncertain state of the financial assets, economic environment, and so on.

The modern design of decision-making under uncertainty is closely related to the application of fuzzy set theory which developed by American scientist L. Zade. Expert evaluates of alternatives for a variety of measure for decision-making can be represented as fuzzy sets or numbers expressed using membership functions. The theory of fuzzy sets has found its application in different fields of mathematics, biology, psychology, linguistics and other application areas.

In this paper we interested in determining of fuzzy modeling in management and other relevant science (Economic, Financial and Ecological) and we would have a vector options to efficiency and environment program.

Keywords: Decision making, Fuzzy factor, Management, Financial, Indicators of the object, Optimal investment, Fuzzy guaranteed result.

Introduction
As noted above, the modern development of decision-making under uncertainty is mainly related to the application of the fuzzy sets theory. The presence in decision-making uncertainty does not allow us to accurately assess the impact of control actions on the objective function. If uncertainty which are exists as in the system itself, so in the observations, can be represented as stochastic processes, so methods of stochastic control are applicable to the such problems. However there is a relatively large class of problems, the solution of which these methods are ineffective (see, for ex. Алтунин А.Е., Семухин М.В., 2002; Mizumoto M., Tanaka K., 1981; Mamdani E.H., Efстаthion H.J., 1985).

Historically, the first and most common is the probabilistic approach to deal with uncertainty. But its use is not always correct, because it requires statistical homogeneity of random events and knowledge of the distribution law, so sometimes introduced nonclassical subjective probability which are not has a partial sense and expresses a person point of view who decide with a deficit of information. Source of uncertainty cannot be random and sometimes can be partially or fully deterministic. At the present time developed quantitative decision-making methods (maximization of expected utility theory of minimax, game theory, etc.) helps to choose the best solution from a set of options only in terms of one particular type of uncertainty or with full certainty. The application of the theory of probability for operating with uncertain values leads to the fact that uncertainty, regardless of its nature, is identified with the accident, while blurry or fuzzy (fuzziness) is the main source of uncertainty in many decision-making processes.

Therefore, the account of uncertainty in solving problems largely changes methods of decision making: the principle of representation of input data and model parameters changes, notion of solving the problem and the optimal solution become ambiguous (see for ex. Tagiyev N.F.,Guliyev R.M.,Mirzayev F.A., 2011).

The system of linear algebraic relations (equations and inequalities) is the simple stand more widely in use mathematical model of most of problems of applied and computational mathematics (see for ex. Dubois D, Prade H., 1980). However, in real problems the values of coefficients and right-hand sides of such may have the indefinite even probabilistic character.

The English term “fuzzy sets” suggested by L.Zadeh [Zadeh L.A., 1965] is visually illustrated by
language examples (almost, not quite and so on) and has interesting applications on sphere of artificial intellect in the processes of construction of mathematical models of real situations.

In this work the problems of identification of unknown characteristics of model in case when the coefficients of linear fuzzy relations include these characteristics. They presents evident interest in connection with problems of control by complex systems, medical diagnostics and much other ones, in which determining factors often have fuzzy characters, and another time, in generally, they are determined by subjective way.

Let give some definitions and significations which are used in present work:

\[ R \] is a widen numerical straight; \( \{ x \mid p(x) \} \) is the set of all \( x \) for which the condition \( p(x) \) is satisfied; the one-point set \( \{ C \} (C \in R) \) we’ll interpret as interval \( [C,C] \), the ends of which are coincided; the fuzzy subset \( \tilde{A} \) in \( R \) we identify (see, for ex. Орловский С.А., 1981) by the totality of well regulated pairs \( \{ x, A(x) \}, x \in R \), where \( A: R \to [0,1] \) is the function of belonging of fuzzy defined set \( \tilde{A} \); and its value \( A(x) \) – by the degree of belonging of element \( x \) to \( \tilde{A} \); the fuzzy number (FN) is a normal fuzzy set \( \tilde{A} \) in \( R \) characterized by that all sets of nonnegative level \( A_x \equiv \{ x \mid A(x) \geq \lambda \} \) \((A[0,1])\) are convex and reserved intervals; \( \tilde{N} \) is family of all FN in \( R \).

**Definition 1.** (see, for ex. Алфальд Г., Хербергер Ю., 1987) By interval number(IN) we understand the finite reserved interval \( \alpha = [\alpha^-, \alpha^+] \) over \( R; I(R) \) is the totality of all IN.

All IN in dependence of their structure may be divided by four groups:

\[ I^+(R) \equiv \{ \alpha \in I(R) \mid \forall x \in \alpha : x > 0 \}; \]
\[ I^+(R) \equiv \{ \alpha \in I(R) \mid \forall x \in \alpha : x < 0 \}; \]
\[ I^0(R) \equiv \{ \alpha \in I(R) \mid \alpha = [\alpha^-, \alpha^+] : \alpha^+ - \alpha^- = 0 \}; \]
\[ I^\pm(R) \equiv \{ \alpha \in I(R) \mid \alpha = [\alpha^-, \alpha^+] : \alpha^- \alpha^+ < 0 \}. \]

Further, let \( \alpha, \beta \in I(R) \) and \( +, -, \cdot, \div; \vee, \wedge, \neg \) are four arithmetical operations, the operation of taking of maximum and minimum accordingly. Then the operation (*) acted as \( \alpha(*)\beta = [\alpha \vee \beta; \alpha \wedge \beta] \) synonymous defined a new IN \( \gamma \) where \((\gamma)\in \{+,-,\cdot,\div;\vee,\wedge,\neg\} \). Really, \( x = f(x,y) = x(*)y \) is a continuous function and, therefore \( \alpha(*)\beta \in I(R) \). It is not difficult to verify that \( \forall \kappa \in R; \alpha, \beta \in I(R) \)

\[ \alpha + \beta = [\alpha^- + \beta^-; \alpha^+ + \beta^+]; \alpha - \beta = [\alpha^- - \beta^+; \alpha^+ - \beta^-]; \]
\[ \alpha \cdot \beta = [\alpha^- \beta^- \wedge \alpha^+ \beta^+ \wedge \alpha^- \beta^+ \wedge \alpha^+ \beta^-; \alpha^- \beta^- \vee \alpha^+ \beta^+ \vee \alpha^- \beta^+ \vee \alpha^+ \beta^-]; \]
\[ \alpha \div \beta = \frac{\alpha^- \beta^- \wedge \alpha^+ \beta^+ \wedge \alpha^- \beta^+ \wedge \alpha^+ \beta^-}{\beta^- \wedge \beta^+ \wedge \beta^- \div \beta^+ \wedge \beta^- \div \beta^+}; \]
\[ \alpha \vee \beta = [\alpha^- \vee \beta^-; \alpha^+ \vee \beta^+]; \alpha \wedge \beta = [\alpha^- \wedge \beta^-; \alpha^+ \wedge \beta^+]; \]
\[ \kappa \alpha \equiv [\kappa^{\alpha^-}, \kappa^{\alpha^+}], \alpha \in \alpha \equiv [\kappa^{\alpha^-}, \kappa^{\alpha^+}], \kappa \geq 0; \]

\[ \text{if } \kappa < 0; \alpha \leq \beta \Rightarrow \alpha^- \leq \beta^- \text{ and } \alpha^+ \leq \beta^+; \alpha \leq \beta \Rightarrow \alpha^- \leq \beta^- \text{ and } \alpha^+ \leq \beta^+. \]

Now all linear relations in IN which have form: \( ax = \alpha \) or \( \alpha x = \beta (\alpha, \beta \in I(R)) \) are known, \( x \in I(R) \) – will be found – by method of their solving may be divided by 4 groups (in all there are \( C_4^4 16 \) relations in each case).

We consider the equation

\[ ax \equiv \beta (\alpha, \beta \in I(R)) \]

(1)

or inequality

\[ ax \leq \beta (\alpha, \beta \in I(R)) \]

(2)

Let \( \forall \gamma \in I(R) \) MaxAbs \( \gamma (\text{MinAbs} \gamma) \) - that end of the interval \( \gamma \) which on absolute value not less (not more) other and \( \forall \alpha, \beta \in I(R) \) we define ordinary numbers \( m, n \) and \( d \):

\[ m = \frac{\text{MaxAbs} \beta}{\text{MaxAbs} \alpha}; \quad n = \frac{\text{MinAbs} \beta}{\text{MinAbs} \alpha}; \quad d = |m| - |n|. \]

On the base of above considered properties of the interval arithmetic the truth of next statements is proved.

**Theorem 1.** The equation (1) has solution then and only then when \( \leq 0 \). Otherwise, this equation has not a solution. If \( \gamma = 0 \), then (1) has infinitely number of solutions and always there is a maximal (with respect to including) element - solution. The uniqueness of the solution of equation (1) is conditioned by \( \gamma < 0 \).
Short explanation of Theorem 1: If the equation (1) has a solution \(( \leq 0)\), the number \(m\), depending on its sign will coincide with one of the ends of the segment solutions \(=-[+]\) (i.e. if \(m < 0 \Rightarrow m = x^-\) and, if \(m > 0 \Rightarrow m = x^+\)). If the solution is unique \((d < 0)\), then the solution of (1) is \([m, l]\) (when \(m < 0\) or \([l, m]\) (if \(m > 0\)), where \(l = \frac{\text{Min Abs } \beta}{\text{Max Abs } \alpha}\). In the case when the equation (1) has an infinite number of solutions, then \(X = [m, x]\) (when \(m < 0 \) and \(m \leq x \leq l\)) or \(X = [x, m]\) (with \(m > 0\) and \(l \leq x \leq m\)). Finally, the case \(d > 0\) will corresponds to the absence of solutions of equation (1).

Consider a few examples:

1) \(\alpha = [-1, 4], \beta = [-2, 10]\). Because \(d = \frac{10}{4} - \frac{2}{1} = 0.5 > 0\), then \(ax = \beta\) no solution.

2) \(\alpha = [-5, 2], \beta = [-15, 6]\). Then \(d = \frac{15}{5} - \frac{6}{2} = 0\), then \(ax = \beta\) has an infinite number of solutions \([x^-, 3]\), where \(l = -1, 2 \leq x^- \leq 3\).

3) \(\alpha = [-5, 2], \beta = [-15, 10]\). Because \(d = -2 < 0\), then the original equation \(ax = \beta\) has a unique solution \(X = [-2, 3]\) (in this case \(l = 3\)).

**Theorem 2.** For solving of inequality (2) it is necessary and sufficiently those relations \(\beta = \text{Max Abs } \beta\) and \(|m| - |n| > 0\) are not satisfied simultaneously.

Indeed, it is not difficult to check that if \(\beta = \text{Max Abs } \beta\) and \(|m| - |n| \leq 0\) inequality (2) always has a solution:

- if \(m > 0 \Rightarrow x^- \in [-\infty, l]\), \(x^+ \in [m, n]\);
- if \(m < 0 \Rightarrow x^- \in [n, m]\), \(x^+ \in [x^-, l]\).

In the case if \(|m| - |n| > 0\), the relation (2) has no solutions. Next, in the case if \(\text{Max Abs } \beta = \beta^+\), then (2) can always be solved:

- when \(m > 0 \Rightarrow x^- \in [k, l]\), \(x^+ \in [x^-, m]\);
- if \(m < 0 \Rightarrow x^- \in [m, x^+], x^+ \in [l, k]\), where \(k = \frac{\text{Max Abs } \beta}{\text{Min Abs } \alpha}\).

Further, these results are applied to approximately (with a sufficiently accuracy) solution of linear fuzzy equations

\[\alpha x + \beta = \gamma\]  
\[(3)\]

or inequality

\[\alpha x + \beta \leq \gamma\]  
\[(4)\]

of more general form, where \(\alpha, \beta, \gamma \in \mathbb{N}\) are known and \(x \in \mathbb{N}\) will be found IN.

As each fuzzy set \(D\) in case when it is FN is quite described by boundary points of reserved intervals \(D_\lambda, (\lambda \in [0, 1])\), then “clear equivalent” of the equation (3) or inequality (4) will be in form

\[\alpha_\lambda + \beta_\lambda = \gamma_\lambda\]  
\[(5)\]

or

\[\alpha_\lambda + \beta_\lambda \leq \gamma_\lambda\]  
\[(6)\]

In other words the equation (3) (inequality (4)) has solution in FN then and only then when equation (5) (inequality (6)) has a solution in IN by all \(\lambda \in [0, 1]\).

Further, in this work the concrete problems of identification which are solved by means of statements of theorem 1 and theorem 2 are given.

**Research methods**

Decision making under uncertainty is very diverse, and its complexity is much superior to similar problems in the deterministic (i.e. in the absence of uncertainty) case.

To formalize most tasks theory of decision making, under conditions of stochastic uncertainty, typically uses probability theory and based on it statistical decision theory and queuing theory.

The successful application of mathematical methods for the analysis of many applications with uncertain parameters can be performed using the methods of interval analysis.

In management decision-maker is often faced with a lot of cases when it isn’t possible to avoid the problem of the uncertainty caused by lack of clarity (fuzzy) goals and (or) restrictions.

A sure step in the formalization and analysis of such decision-making problems (as well as the
application of information technology in the "non-traditional" or "humanitarian" fields, such as economics, medicine, sociology), and in building mathematical, environmental, economic, and so etc. models of specific processes, apparatus of fuzzy set theory is considered a fairly new area of applied mathematics, associated with the name of a prominent mathematician of L.A.Zadeh.

Typically, the main goal of any business is profit. In the case of construction or operation of any financial entity there is a problem of its profitability, because if its yield is below the average interest rate then its existence is meaningless in terms of profit. In financial entities (such as banks, investment funds, insurance companies, brokerage, dealer firms, etc.) the basic moments are the income from the placement and the costs in the form of payments on borrowed funds.

The most important task of commercial banks is also getting profit. For this purpose they use a variety of features, including expanding credit operations, increase services to the population. However, it is important to maintain the liquidity of each bank, which usually refers to the ability of the bank promptly and fully repay its obligations to the customers, other banks, etc.

The combination of the desire to increase profits and liquidity support should be an important landmark in the banks. However, this is not consistently enforced.

For better grounding decision-making to attract and place money it is suggested to consider the general methodology for calculating key performance indicators of the bank and their prediction. Key indicators are derived from the main purpose of the bank - attraction and allocation of funds. The main indicator for the raised funds is the average rate of interest on borrowed funds, the main indicator for allocated funds is the yield of active operations (calculated as a percentage).

Naturally, for the calculation of the indicators it is necessary to have information collected during the period. In banking, today it is not difficult, because all banks have automated systems to ensure the operations of the bank which accumulate information from the inception of the bank (the system of the bank) Thus with the availability of data there are no difficulties with their processing. However, as noted above, the data (or part of the data) are usually unclear, since they are mainly determined by the subjective (expert) way.

In the case of fuzzy methods, for example, in the financial business, as opposed to existing planning and management techniques, it is possible to use different views of the active persons engaged in the planning and decision-makers.

Analysis or Discussion

The problem of studying the relationship of economic indicators is one of the major problems of economic analysis. Therefore, any management activity is to regulate the economic variables, and it should be based on knowledge of how these variables affect other variables that are crucial to the decision-making policy. Thus, in a market economy it is impossible to directly regulate the rate of inflation, but it can be influenced by means of fiscal and monetary policy. Therefore, in particular, the relationship between the money supply and the price level should be studied.

This work is dedicated to discussion of the basic principles of modeling with fuzzy uncertainty. For example, in determining the coefficients $A_i$ of the corresponding regression linear model ($X_i$ are indicators of the object), the coefficients-parameters of the model are naturally identified with the fuzzy sets (in most cases - fuzzy numbers), and the simulation should be performed for the fuzzy phenomena and systems:

$$ Y = A_1X_1 + \ldots + A_nX_n. $$

The solution is obtained in a fuzzy form corresponding to fuzzy set information.

Note that the investment in the real economy the banks and other investors should reasonably consider not only the investment program, but also the financial, industrial, economic and socio-economic activities of the company. Therefore, the decision-maker (DM) is interested in the study of the correlations of investments with other areas, and, above all, with the financing and production.

The ways of making investment and financial program solutions in definite situations (i.e. when the future income and expenses associated with the implementation of the project are assumed to be known) can be combined into a group of models to determine (see for example Андрейчиков А.В., Андрейчикова О.Н., 2000):...
- optimal investment program for a given individual investment object of a production program with a given production budget
- simultaneously both an investment and financial programs for a given production program for an individual investment property;
- simultaneously optimal investment and financial program for the given financial resources and with the involvement of the various alternatives to the model of finance. However, the transition to market-based economic relations leads to a significant expansion of investment activities through the creation and development along with the goods and services markets of the capital market, which is a collection of various financial markets. Therefore, for a large part of investment projects future revenues and costs associated with the implementation of the project, cannot be determined unambiguously, and investors in their decisions are often faced with the uncertainty of their evaluation. The reasons for this are due to the circumstances of both the essence of the market economy (in which the future performance of an investment or other business activities depend strongly on market conditions experiencing the influence of many factors that do not depend on the efforts of investors), and the fact that the economic phenomena and processes, as a rule, are exposed to a sufficient number of non-economic factors (climatic and environmental conditions, political, social, etc.), which cannot always be an accurate assessment and prediction.

At present fuzzy logic theory has become very popular for practical applications in many fields of science. In the area of decision-making on the basis of this theory a wide range of different methods has developed. In particular, for forecasting and other planning problems in the business on the basis of data received from the experts, it is necessary to build fuzzy regressive nonlinear model. In this case, it is appropriate to use fuzzy sets as the undetermined coefficients of the model.

Among the areas of wide application of fuzzy set theory as a special place is occupied by problem of mathematical programming with fuzzy parameter values and (or) restrictions. Finally, we consider an optimization-management model combining production program with a finite number of production structures with the environmental factor (i.e. the part of output is spent on environmental protection) [Tagiyev N.F., Guliyev R.M., Mirzayev F.A., 2010]:

\[
(C, Y) \rightarrow \max,
(Q, Y) \leq R, \\
Y \geq 0.
\]

In this model:
- \(Y\) – a vector of environmental program options;
- \(C\) – a vector of efficiency options;
- \(Q\) – a matrix of unit costs of the program versions;
- \(R\) – a vector of limit for environmental costs.

With advance planning it is possible that components of the vectors \(C\), \(Q\) and \(R\) are appointed by the coordinating center and some deviations from the "policy" value are assumed. As a result, the values of the components of these vectors parametrically depend on the degree of permissibility.

Obviously, with such an interpretation the above components of the vectors will be fuzzy sets of allowable values for each efficiency variant, unit costs and limit for environmental costs. The resulting fuzzy linear programming problem can be solved by means of the theory of fuzzy sets.

In the majority of such problems fuzzy guaranteed result is better in terms of optimization than normal (after the usual sets are subsets of the corresponding fuzzy sets).

**Conclusion**

In our opinion, a difficult challenge today is to choose a suitable method or software to support different processes of decision-making. Therefore, it is especially important to conduct a comparative analysis (under the condition that there is uncertainty of different kinds) of specific methods and recommendations for their use.

**References**