WEIBULL DETERIORATING ITEMS OF PRICE DEPENDENT DEMAND OF QUADRATIC HOLDING FOR INVENTORY MODEL

S. Mohan Prabhu  
Research and Development Centre,  
Bharathiar University,  
Coimbatore-641 046.  
Lecturer, Muthayammal College of Arts and Science,  
Rasipuram, Namakkal-63740

Dr. P. Puthiyayagam  
Associate Professor,  
Subbalakshmi Lakshmipathi College of Science,  
Madurai-625 022.

ABSTRACT

In this paper discuss with development of an inventory model when the deterioration rate follows Weibull distributions. Here it is assumed that demand rate is a function of selling price and holding cost. With shortage and without shortage both cases have been taken care of in developing in the inventory models. Shortages are completely backlogged whenever they are allowed. The conclusions are illustrated with help of the numerical examples. The sensitivity analysis for the model has been performed to study the effect changes of the values of the Inventory model.

Keywords: Deteriorating items; EOQ model; Price dependent demand; Shortage; Weibull distribution.
1. Introduction

The control and maintenance inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over time. In reality, some of the items are either damaged or decayed or affected by some other factors and is not in a perfect condition to satisfy the demand. Food items, drugs, pharmaceuticals, radioactive substances are examples of such items where deterioration can take place during the normal storage period of the commodity and consequently this loss must be taken into account when analyzing the system. So decay or deterioration of physical goods in stock is a very realistic feature and researchers felt the necessity to use this factor into consideration in developing inventory models.

Chakraborty and Chaudhuri [4], Inventory models with a time dependent rate of deterioration were considered by Covert and Philip [6], developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by Ghare and Schrader [10], Burwell [2] developed economic lot size model for price-dependent demand under quantity and freight discounts, Shah and Jaiswal, [20], Aggarwal [1], Some of the significant recent work in this field have been done by Chung and Ting [5], Dave and Patel [7], Deb and Chaudhuri [8], Giri and Chaudhuri [11], Chakraborty et al., [3], Fujiwara [9], Jalan and Chaudhuri [15], Goh [12], Hariga and Benkherouf [14], Jalan et al., [16], Muhlemann and Valtis Spanopoulos [19], Mishra [17], Hariga [13], At the beginning, demand rate were assumed to be constant which is in general likely to be time dependent and stock dependent. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal, Bhunia and Maiti [18], Weiss [24], Tripathy and Mishra [21], Inventory model with price and time dependent demand was developed by You,S.P.,[25], Wee [23], In general holding cost is assumed to be known and constant. But in realistic condition holding cost may not always be constant. So several researchers like Van der Veen [22], and considered various functions to describe holding cost and etc.

In this paper, we have developed generalized EOQ model for deteriorating items where deterioration rate follows two-parameter Weibull distribution and holding cost are expressed as linearly increasing functions of time and demand rate is considered to be a function of selling price. For the model where shortages are allowed they are completely backlogged. If the holding cost is non-linear, a similar analysis is presented for the trend functions.

They have analyzed with shortage and without shortage inventory model for a linear trend \( h(t) = h + \delta t \), where \( \delta > 0, h > 0 \) in holding cost and it is extended by taking the holding cost \( h(t) = a + bt + ct^2 \), a, b, c > 0. Here we have considered both the case of with shortage and without shortage in developing the model.

2. Assumptions and Notations

The fundamental assumptions of the model are as follows:

a) The demand rate is a function of selling price.

b) Shortages, whenever allowed, are completely backlogged.

c) The deterioration rate is proportional of fixed time.

d) Holding cost \( h(t) \) per item per time-unit is time assumed to be

\[
h(t) = a + bt + ct^2, \quad \text{where } a > 0, b > 0, c > 0
\]

Considered the case of with shortage and without shortage.

e) Replenishment is instantaneous and lead time is zero.

f) \( T \) is the length of the cycle.
g) The order quantity in one cycle is $q$.
h) $A$ is the cost of placing an order.
i) The selling price per unit item is $p$.
j) $C$ is the unit cost of an item.
k) The inventory holding cost per unit per unit time is $h(t)$.
l) $C_1$ is the shortage cost per unit per unit time.
m) The deterioration of units follows the two parameter Weibull distribution (say) 
$$\theta(t) = \alpha t^{\beta-1}$$ where $0 < \alpha < 1$ is the scale parameter and $\beta > 0$ is the shape parameter.
n) During time $t_1$, inventory is depleted due to deterioration and demand of the item.

At

time $t_1$ the inventory becomes zero and shortages start occurring.
o) Selling price $p$ follows an increasing trend, and demand rate possess the negative derivative through out its domain where demand rate is 
$$f(p) = (a_1 - p) > 0$$

3. Mathematical formulation and solution

Let $Q(t)$ be the inventory level at time $t$ ($0 \leq t \leq T$). The differential equations to describe instantaneous state over $(0, T)$ are given by,

$$\frac{dQ(t)}{dt} + \alpha ft^{\beta-1} Q(t) = -(a_1 - p)$$

$$0 \leq t \leq t_1 \quad \rightarrow (1)$$

$$\frac{dQ(t)}{dt} = -(a_1 - p), \quad t_1 \leq t \leq T \quad \rightarrow (2)$$

With $Q(t) = 0$ at $t = t_1$

Solving equation (1) and equation (2) and neglecting higher powers of $\alpha$, we get In equation (1),

$$Q(t)e^{\alpha t^{\beta} dt} = \int -(a_1 - p)e^{\alpha t^{\beta} dt} + c$$

$$Q(t)e^{\alpha t^{\beta}} = -(a_1 - p) \int t^{\beta} e^{\alpha t^{\beta} dt} + c$$

$$Q(t)e^{\alpha t^{\beta}} = -(a_1 - p) \int t^{\beta} e^{\alpha t^{\beta}} dt + c$$

$$Q(t)e^{\alpha t^{\beta}} = -(a_1 - p) \left[ t + \frac{at^{(\beta + 1)}}{(\beta + 1)} \right]$$

$$Q(t)e^{\alpha t^{\beta}} = -(a_1 - p) \left[ t + \frac{at^{(\beta + 1)}}{\beta + 1} - t - \frac{at^{(\beta + 1)}}{\beta + 1} \right]$$

$$Q(t)e^{\alpha t^{\beta}} = -(a_1 - p) \left[ t - t_1 - \frac{a}{\beta + a} \left( t^{(\beta + 1)} - t_1^{(\beta + 1)} \right) \right]$$
\[ Q(t) = -(a_1 - p)e^{-at} \left[ (t_0 - t) - \frac{\alpha}{(\beta + 1)} \left( t^{(\beta+1)} - t_1^{(\beta+1)} \right) \right] \]

\[ Q(t) = -(a_1 - p)(1 - at^\beta) \left[ (t_0 - t) - \frac{\alpha}{(\beta + 1)} \left( t^{(\beta+1)} - t_1^{(\beta+1)} \right) \right] \]

\[ Q(t) = -(a_1 - p)(1 - at^\beta) \left[ (t - t_1) - \frac{\alpha}{(\beta + 1)} \left( t^{(\beta+1)} - t_1^{(\beta+1)} \right) \right]; \quad 0 \leq t \leq t_1 \]

and in equation (2)
\[ \frac{dQ(t)}{dt} = -(a_1 - p), t_1 \leq t \leq T \]

Taking integration on both sides,
\[ \int \frac{dQ(T)}{dt} \, dt = \int -(a_1 - p) \, dt \]

\[ Q(t) = -(a_1 - p) \int_0^t dt \]

\[ Q(t) = -(a_1 - p)[t]_0^t \]

\[ Q(t) = -(a_1 - p)(t - t_1), \quad 0 \leq t_1 \leq t \]

3.1 Now stock loss due to deterioration

\[ D = (a_1 - p) \int_0^t \left[ e^{at} \right] dt - (a_1 - p) \int_0^t dt \]

\[ = (a_1 - p) \int_0^t \left[ 1 + at^\beta \right] dt - (a_1 - p) \int_0^t dt \]

\[ = (a_1 - p) \left[ t + \frac{at(\beta + 1)}{(\beta + 1)} \right]_0^t - (a_1 - p)[t]_0^t \]

\[ = (a_1 - p) \left[ t_1 + \frac{at_1(\beta + 1)}{(\beta + 1)} \right] - (a_1 - p)[t_1] \]

\[ = (a_1 - p) \left\{ \frac{at_1(\beta + 1)}{(\beta + 1)} \right\} \]

\[ D = (a_1 - p) \left\{ \frac{at_1(\beta + 1)}{(\beta + 1)} \right\} \]

\[ q = D + \int_0^t (a_1 - p) \, dt \]

\[ = (a_1 - p) \left\{ \frac{at_1(\beta + 1)}{(\beta + 1)} \right\} + (a_1 - p) \int_0^t \, dt \]

\[ = (a_1 - p) \left\{ \frac{at_1(\beta + 1)}{(\beta + 1)} \right\} + (a_1 - p)[t]_0^t \]
$$= (a_i - p) \left( \frac{\alpha t^{(\beta+1)}}{\beta + 1} \right) + (a_i - p)[T]$$

$$q = (a_i - p) \left\{ \frac{\alpha t^{(\beta+1)}}{\beta + 1} + T \right\} \quad \rightarrow \quad (3)$$

Holding cost is,

$$H = \int_0^t (a + bt + ct^2)e^{-\alpha t} \left\{ b t, (a_i - p)e^{\alpha u^p} \right\} dt$$

Neglecting higher powers of \( a \), we get

$$H = (a_i - p) \left[ \int_0^t (a + bt + ct^2)(1 - \alpha t^p) \left\{ b t, (1 + \alpha u^p) \right\} dt \right]$$

$$H = (a_i - p) \left[ \int_0^t (a + bt + ct^2)(1 - \alpha t^p) \left[ u + \alpha \frac{t^{(\beta+1)}}{\beta + 1} \right] dt \right]$$

$$H = (a_i - p) \left[ \int_0^t (a + a\alpha t^p + bt - b\alpha t^p + ct^2 - c\alpha t^p + c\alpha t^p) \left[ t + \alpha \frac{t^{(\beta+1)}}{\beta + 1} - t - \alpha \frac{t^{(\beta+1)}}{\beta + 1} \right] dt \right]$$

$$H = (a_i - p) \left[ \int_0^t \left[ t_i + \alpha \frac{t^{(\beta+1)}}{\beta + 1} - t - \alpha \frac{t^{(\beta+1)}}{\beta + 1} \right] dt \right]$$

$$H = (a_i - p) \left[ \int_0^t \left[ t_i + \alpha \frac{t^{(\beta+1)}}{\beta + 1} - t - \alpha \frac{t^{(\beta+1)}}{\beta + 1} \right] dt \right]$$

$$H = (a_i - p) \left[ \int_0^t \left[ t_i + \alpha \frac{t^{(\beta+1)}}{\beta + 1} - t - \alpha \frac{t^{(\beta+1)}}{\beta + 1} \right] dt \right]$$
\( H = (a_1 - p) \left[ a t_1 + \frac{a \alpha t_1 (\beta + 2)}{\beta + 1} - \frac{a \alpha t_1 (\beta + 2)}{2 (\beta + 1) (\beta + 2)} - \frac{a \alpha t_1 (\beta + 2)}{(\beta + 1)} - \frac{a \alpha t_1 (\beta + 2)}{(\beta + 1)^2} \right. \\
+ \frac{a \alpha t_1 (\beta + 2)}{\beta + 2} - \frac{a \alpha t_1 (\beta + 2)}{2 (\beta + 1)} + \frac{b t_1^2}{2} + \frac{b \alpha t_1 (\beta + 3)}{2 (\beta + 1)} - \frac{b t_1^2}{3} - \frac{b \alpha t_1 (\beta + 3)}{(\beta + 1) (\beta + 3)} \\
- \frac{b \alpha t_1 (\beta + 3)}{(\beta + 1) (\beta + 2)} + \frac{b \alpha t_1 (\beta + 3)}{\beta + 3} + \frac{b \alpha t_1 (\beta + 3)}{(\beta + 1) (2 \beta + 3)} \right. \\
+ \frac{ct_1^4}{3} + \frac{c \alpha t_1 (\beta + 4)}{3 (\beta + 1)} \\
- \frac{ct_1^4}{4} - \frac{c \alpha t_1 (\beta + 4)}{(\beta + 1) (\beta + 4)} - \frac{c \alpha t_1 (\beta + 4)}{(\beta + 1) (\beta + 3)} + \frac{c \alpha t_1 (\beta + 4)}{(\beta + 4)} \\
\left. + \frac{c \alpha t_1 (\beta + 4)}{(\beta + 1) (2 \beta + 4)} \right] \\
\]

\( H = (a_1 - p) \left[ a \left\{ t_1^2 \left(1 - \frac{1}{2}\right) + \frac{\alpha t_1 (\beta + 2)}{\beta + 1} \left(1 - \frac{1}{\beta + 1}\right) - \frac{\alpha t_1 (\beta + 2)}{(\beta + 1)^2} \left(1 - \frac{1}{2}\right) \right\} \\
+ b \left\{ t_1^2 \left(1 - \frac{1}{2}\right) + \frac{\alpha t_1 (\beta + 3)}{\beta + 1} \left(1 - \frac{1}{\beta + 3}\right) - \alpha t_1 (\beta + 3) \left(\frac{1}{\beta + 2} - \frac{1}{\beta + 3}\right) \right\} \\
- \frac{\alpha t_1 (\beta + 3)}{(\beta + 1) (2 \beta + 3)} + c \left\{ t_1^2 \left(1 - \frac{1}{2}\right) + \frac{\alpha t_1 (\beta + 4)}{\beta + 1} \left(1 - \frac{1}{\beta + 4}\right) \right\} \right] \\
\]

\( H = (a_1 - p) \left[ a \left\{ t_1^2 + \frac{\beta \alpha t_1 (\beta + 2)}{(\beta + 1) (\beta + 2)} - \frac{\alpha t_1 (\beta + 2)}{2 (\beta + 1)^3} \right\} + b \left\{ t_1^2 + \frac{\alpha t_1 (\beta + 3)}{2 (\beta + 3)} \right\} \\
- \frac{\alpha t_1 (\beta + 3)}{(\beta + 2) (\beta + 3)} - \frac{\alpha t_1 (\beta + 3)}{(\beta + 2) (2 \beta + 3)} + c \left\{ t_1^4 + \frac{\alpha t_1 (\beta + 4)}{12} - \frac{\alpha t_1 (\beta + 4)}{3 (\beta + 4)} - \frac{\alpha t_1 (\beta + 4)}{(\beta + 3) (\beta + 4)} \right\} \right] \]

3.2 Now shortage cost during the cycle,
\[ S = - \int_{t_1}^{T} \left\{ -(a_1 - p)(t - t_1) \right\} dt \]
\[ S = (a_1 - p) \int_{t_1}^{T} (t - t_1) dt \]
\[ S = (a_1 - p) \left[ \frac{t^2}{2} - t_1 t \right]_{t_1}^{T} \]
\[ S = (a_1 - p) \left[ \frac{T^2}{2} - T t_1 - \frac{T^2}{2} + t_1^2 \right] \]
\[ S = (a_1 - p) \left[ \frac{T^2}{2} - T t_1 + \frac{t_1^2}{2} \right] \]
From (3), (4) and (5) Total profit per unit time is given by
\[
\begin{align*}
\mathcal{P}(T, t_1, p) &= p(a_1 - p) - \frac{1}{T} (A + cq + H + c_1 S) \\
&= p(a_1 - p) - \frac{1}{T} 
\left[ A + C \left( (a_1 - p) \left( \frac{\alpha_{1,1}^{(\beta+1)}}{(\beta + 1)} + T \right) \right) + (a_1 - p) \left( \frac{t_1^2}{2} \right) \\
&\quad + \frac{\beta \alpha_{1,1}^{(\beta+2)}}{(\beta + 1)(2\beta + 2)} \left( \frac{t_1^2}{(2\beta + 2)} \right) \right] + b \left[ \frac{t_1^2}{6} + \frac{\alpha_{1,1}^{(\beta+3)}}{2(\beta + 3)} - \frac{\alpha_{1,1}^{(\beta+3)}}{(\beta + 2)(\beta + 3)} \right] \\
&\quad - \frac{\alpha_{2,1}^{(2\beta+3)}}{(\beta + 2)(2\beta + 3)} \left( \frac{t_1^4}{12} + \frac{\alpha_{1,1}^{(\beta+4)}}{3(\beta + 4)} - \frac{\alpha_{1,1}^{(\beta+4)}}{(\beta + 3)(\beta + 4)} \right) \\
&\quad - \frac{\alpha_{2,2}^{(2\beta+4)}}{(\beta + 3)(2\beta + 4)} \left( \frac{t_1^4}{2(\beta + 3)} - \frac{\alpha_{1,1}^{(\beta+4)}}{(\beta + 2)(\beta + 3)} \right) + c_1 \left( \frac{1}{2} (a_1 - p)(T - t_1)^2 \right) \right] 
\right) 
\end{align*}
\]

Let \( t_1 = \gamma T \), \( 0 < \gamma < 1 \)

Hence we get the profit function,
\[
\begin{align*}
\mathcal{P}(T, p) &= p(a_1 - p) - \frac{1}{T} \left[ A + C \left( (a_1 - p) \left( \frac{\alpha_{1,1}^{(\beta+1)}}{(\beta + 1)} + T \right) \right) \right] \\
&\quad + (a_1 - p) \left[ \frac{\gamma^2 T^2}{2} + \frac{\beta \alpha_{1,1}^{(\beta+2)}}{(\beta + 1)(2\beta + 2)} \left( \frac{t_1^2}{(2\beta + 2)} \right) \right] + b \left[ \frac{\gamma^2 T^2}{6} + \frac{\alpha_{1,1}^{(\beta+3)}}{2(\beta + 3)} - \frac{\alpha_{1,1}^{(\beta+3)}}{(\beta + 2)(\beta + 3)} \right] \\
&\quad + \frac{\alpha_{1,1}^{(\beta+3)}}{2(\beta + 3)} \left( \frac{t_1^4}{(2\beta + 2)} \right) - \frac{\alpha_{1,1}^{(\beta+4)}}{(\beta + 3)(2\beta + 4)} \left( \frac{t_1^4}{3(\beta + 4)} \right) + c_1 \left( \frac{1}{2} (a_1 - p)T^2 (1 - \gamma)^2 \right) 
\right] 
\end{align*}
\]

Our objective is to Maximize the profit function \( \mathcal{P}(T, p) \). The necessary conditions for maximizing the profit are
\[
\frac{\partial \mathcal{P}(T, p)}{\partial p} = 0
\]

We get,
\[
= a_i - 2p - \frac{1}{T} \left[ - C \left( \frac{\alpha_{1,1}^{(\beta+1)}}{(\beta + 1)} + T \right) \right] - \left( \frac{\gamma^2 T^2}{2} + \frac{\beta \alpha_{1,1}^{(\beta+2)}}{(\beta + 1)(\beta + 2)} \right)
\]
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Research Journal of Economics & Business Studies

\[-\frac{\alpha^2 \gamma^2}{2(\beta+1)^2} \left( T(\beta+2) \right)^2 \] + \left[ \frac{\gamma^3 T^3}{6} + \frac{\alpha \gamma (\beta+3)}{2(\beta+3)} - \frac{\alpha \gamma (\beta+3)}{(\beta+2)(\beta+3)} \right] \] 
\[-\frac{\alpha^2 \gamma^2}{(\beta+2)(2\beta+3)} \right] + C \left[ \frac{\gamma^4 T^4}{12} + \frac{\alpha \gamma (\beta+4)}{3(\beta+4)} - \frac{\alpha \gamma (\beta+4)}{(\beta+3)(\beta+4)} \right] \right] \right] \right] 
\[-\frac{\alpha^2 \gamma^2}{(\beta+3)(2\beta+4)} \right] \right] - \frac{C}{2} T^2 (1-\gamma)^2 \right] = 0 \] 
\[= a_1 - 2p + \frac{1}{T} C \left[ \frac{\alpha (\gamma T)^{\beta+1}}{(\beta+1)} + T \right] + a \left[ \frac{(\gamma T)^2}{2} + \frac{\beta \alpha (\gamma T)^{\beta+2}}{(\beta+1)(\beta+2)} \right] \] 
\[+ b \left[ \frac{(\gamma T)^3}{6} + \frac{\alpha (\gamma T)^{\beta+3}}{2(\beta+3)} - \frac{\alpha (\gamma T)^{\beta+3}}{(\beta+2)(\beta+3)} \right] \] 
\[+ C \left[ \frac{(\gamma T)^4}{12} + \frac{\alpha (\gamma T)^{\beta+4}}{3(\beta+4)} - \frac{\alpha (\gamma T)^{\beta+4}}{(\beta+3)(\beta+4)} \right] \] 
\[+ \frac{C}{2} T^2 (1-\gamma)^2 \right] = 0 \] 
\[\rightarrow (8) \]

From Equation (8),
We can calculate the optimum values of \( p^* \)
\[p^* = \frac{1}{2} \left[ a_1 + \frac{1}{T} C \left[ \frac{\alpha (\gamma T)^{\beta+1}}{(\beta+1)} + T \right] + a \left[ \frac{(\gamma T)^2}{2} + \frac{\beta \alpha (\gamma T)^{\beta+2}}{(\beta+1)(\beta+2)} \right] \] 
\[-\frac{\alpha^2 (\gamma T)^{2\beta+2}}{2(\beta+1)^2} \right] + b \left[ \frac{(\gamma T)^3}{6} + \frac{\alpha (\gamma T)^{\beta+3}}{2(\beta+3)} - \frac{\alpha (\gamma T)^{\beta+3}}{(\beta+2)(\beta+3)} \right] \] 
\[-\frac{\alpha^2 (\gamma T)^{2\beta+3}}{(\beta+2)(2\beta+3)} \right] + C \left[ \frac{(\gamma T)^4}{12} + \frac{\alpha (\gamma T)^{\beta+4}}{3(\beta+4)} - \frac{\alpha (\gamma T)^{\beta+4}}{(\beta+3)(\beta+4)} \right] \] 
\[-\frac{\alpha^2 (\gamma T)^{2\beta+4}}{(\beta+3)(2\beta+4)} \right] + \frac{C}{2} T^2 (1-\gamma)^2 \right] \] 

And the optimal value \( P^*(T, p) \) of the average net profit is determined by (7) provided they satisfy the sufficiency conditions for maximizing \( P^*(T, p) \) is,
\[\frac{\partial^2 P(T, p)}{\partial p^2} < 0 \text{ at } p = p^* \]
If the solutions obtained from equation (8) do not satisfy the sufficiency condition. We conclude that no feasible solution will be optimal for the set of parameter values taken to solve equation (8).

Such a situation will imply that the parameter values are inconsistent and there is some error in their estimation.

4. Numerical Example

4.1 Case-I (With Shortages)
Example-1
Let A=200, \( a_1 = 100 \), C=20, \( a=0.2 \), b=0.4, \( C=0.6 \), \( c_1 =1.2 \), \( \alpha=0.1 \), \( \beta=0.3 \), \( \gamma=0.5 \), and \( T=3.14165 \).
Based on these input data, the computed outputs are as follows:
\[
\begin{align*}
p^* (p) &= 1472.5926 \\
p^* &= 60.8056 \\
t_i^* &= 1.570825 \\
q^* &= 128.5582
\end{align*}
\]

4.2 Case-II (Without Shortages)
Example-1
Let A=200, \( a_1 =100 \), C=20, \( a=0.2 \), b=0.4, c=0.6, \( \alpha=0.1 \), \( \beta=0.3 \), \( \gamma=0.5 \), and \( T=3.92398 \).
Based on these input data, the computed outputs are as follows:
\[
\begin{align*}
p^* (p) &= 1495.1507 \\
p^* &= 60.6793 \\
t_i^* &= 1.96199 \\
q^* &= 161.5579
\end{align*}
\]

Table-1

<table>
<thead>
<tr>
<th>Changing Parameter</th>
<th>% Change in System</th>
<th>Change in ( p^* )</th>
<th>Change in ( q^* )</th>
<th>Change in ( p^* (p) )</th>
</tr>
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<tbody>
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<td>44.9027</td>
<td>137.5666</td>
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<td>1473.8782</td>
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</tbody>
</table>
We study from above Table-1 reveals the following
(i) Increase in the values of either of the parameter $a_i$, will result in increase of $P^*(p), p^* and q^*$. 
(ii) Decrease in the values of either of the parameter $a_i$, will result in decrease of $P^*(p), p^* and q^*$. 
(iii) Increase in the values of either of the parameter $\alpha$, will result in decrease of $P^*(p)$ but increase $p^* and q^*$. 
(iv) Decrease in the values of either of the parameter $\alpha$, will result in increase of $P^*(p)$ but decrease $p^* and q^*$. 
(v) Increase in the values of either of the parameter $\beta$, will result in increase of $P^*(p)$ but increase $p^* and q^*$. 
(vi) Decrease in the values of either of the parameter $\beta$, will result in decrease of $P^*(p)$ but increase $p^* and q^*$. 

We study from above Table-2 reveals the following
(i) Increase in the values of parameter $a$, will result in increase of $P^*(p), p^* and q^*$. 
(ii) Decrease in the values of parameter $a$, will result in decrease of $P^*(p), p^* and q^*$. 
(iii) Increase in the values of parameter $\alpha$, will result in decrease of $P^*(p)$ but increase $p^* and q^*$. 
(iv) Decrease in the values of parameter $\alpha$, will result in increase of $P^*(p)$ but decrease $p^* and q^*$. 
(v) Increase in the values of parameter $\beta$, will result in increase of $P^*(p)$ but increase $p^* and q^*$. 
(vi) Decrease in the values of parameter $\beta$, will result in decrease of $P^*(p)$ but increase $p^* and q^*$. 

5. Sensitivity Analysis and Conclusion
we have developed deterministic inventory model for deteriorating items for with shortage and without shortage cases. The deterministic demand rate is assumed to be a
function of selling price. Whenever shortage are allowed and they are completely backlogged and holding cost is assumed here to be time varying. We can make a good comparative study between the results of the with-shortage case and without shortage case.

In the numerical examples, it is found that the optimum average profit in without-shortage case is more than that of the shortage case.

To study the effects of changes of the parameters on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical example given above. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 20% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in Table-1 and Table-2 for with shortage case and without shortage case respectively.

REFERENCES


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